POINCARÉ 2-GROUP AND THE SPINCUBE MODEL OF QUANTUM GRAVITY

Marko Vojinović
Institute of Physics, University of Belgrade

in collaboration with

Aleksandar Miković
Lusofona University and GFMUL, Portugal

THE PROBLEM OF QUANTUM GRAVITY

Why quantize gravity?

- same reasons as electrodynamics (two-slit experiment, hydrogen atom, . . .)
- resolution of singularities (black holes, Big Bang, . . .)
- black hole information paradox (nonunitary evolution??!!)
- theoretical and aesthetical reasons. . .

How to quantize gravity?

- perturbation theory does not work (nonrenormalizability of gravity). . .
- almost zero experimental results to guide us. . .
- . . . we have a problem!

NONRENORMALIZABILITY OF GRAVITY

Perturbative quantization idea

- expand the metric around flat spacetime,

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x),$$

- use it to expand the Einstein-Hilbert action,

$$S_{EH} = \int d^4x \sqrt{-g}R = \int d^4x \, h_{\mu\nu}\Box h^{\mu\nu} + h^3 + h^4 + \ldots,$$

- obtain a theory for self-interacting massless spin-2 field in flat spacetime,
- quantize in analogy to Yang-Mills theories.

NONRENORMALIZABILITY OF GRAVITY

However, the resulting theory is nonrenormalizable:

- tree-level Feynman diagrams are finite,
- one-loop diagrams require a counterterm — remove it by renormalizing $g_{\mu\nu}$,
- two-loop diagrams require a counterterm of type

$$c_2 \frac{1}{\varepsilon^2} R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}, \qquad (\varepsilon \to 0)$$

  which **cannot be removed by renormalization**.

- each higher-loop diagram requires another nonrenormalizable counterterm.

The theory contains infinitely many coupling constants!

The theory loses its predictive power — each choice of coupling constants fixes a different theory of gravity!

THE PROBLEM OF QUANTUM GRAVITY

$$S = S_{EH} + \int d^4x \, c_2 \mathcal{L}_2 + c_3 \mathcal{L}_3 + \ldots$$

The problem of quantizing gravity "reduces" to

inventing a SET OF FIRST PRINCIPLES that Nature obeys

such that all coupling constants $c_1, c_2, \ldots$ are fixed and can be calculated.

Mainstream candidate approaches:

- String theory
- Noncommutative geometry
- Loop quantum gravity
- Causal dynamical triangulations
- Causal set theory
- Doubly special relativity

… and so on…

There is no experimental data at the Planck scale, to distinguish between these ideas.

LOOP QUANTUM GRAVITY

The idea of LQG

- Wilson loops are chosen as basic degrees of freedom,
- formalized as "spin network states",
- canonically quantized.

Achievemenets

- nonperturbative quantization of GR,
- kinematic sector of the theory well-defined,
- lengths, areas and volumes of space quantized!

Drawbacks

- dynamics described only in principle,
- no proof of semiclassical limit,
- very limited possibility for calculations.

SPINFOAM MODELS

The idea in brief

- build up on canonical LQG (use the same degrees of freedom, construct the same structure of the Hilbert space, etc.),
- discretize spacetime into 4-simplices,
- perform covariant quantization, by providing a definition for the gravitational path integral,

$$Z = \int \mathcal{D}g_{\mu\nu} \exp\left(iS_{EH}[g_{\mu\nu}]\right),$$

- use this definition to calculate expectation values for all interesting observables as in quantum field theory.

SPINFOAM MODELS

The idea in a bit more detail

(1) rewrite GR action using the Plebanski formalism:

$$S = \int B_{ab} \wedge R^{ab} + \phi^{abcd} B_{ab} \wedge B_{cd},$$

(2) quantize the BF sector by (a) triangulating the spacetime manifold, (b) defining the path integral

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \exp\left[i \sum_{\Delta} B_{\Delta} R_{\Delta} \right] = \ldots = \sum_{\Lambda} \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v),$$

where Λ are irreducible representations of $SO(3,1)$, while A_2 and A_4 are chosen such that Z is a topological invariant;

(3) enforce the "Plebanski constraint" by projecting the representations from $SO(3,1)$ to $SU(2)$, and by redefining the vertex amplitude A_4

⇒ obtain a non-topological path integral definition of the theory, with local degrees of freedom.

SPINFOAM MODELS

Main achievements

- well-defined nonperturbative quantum theory of gravity,
- both kinematical and dynamical sectors under control,
- can have a proper semiclassical limit,
- predicts the values of the counterterm coupling constants.

Main drawbacks

- geometry is “fuzzy” at the Planck scale,
- has many different semiclassical limits,
- matter coupling is problematic,
- hard to extract any results.

The reason for these drawbacks: tetrads are not explicitly present in the action!

POINCARÉ GROUP

Properties of the Poincaré group:

- $P(4) = \mathbb{R}^4 \ltimes SO(3,1)$
- Lorentz group has a connection 1-form ω which transforms as a gauge potential

$$\omega \to g^{-1}\omega g + g^{-1}dg, \qquad (g : \mathcal{M}_4 \to SO(3,1))$$

- one can introduce line holonomies

$$g_l(\omega) = \exp \int_l \omega$$

- 4-translation group has a tetrad 1-form e which **does not** transform as a gauge potential! ($\mathbb{I} - e$ does)
- one can associate a BF action to the Lorentz group,

$$S = \int B_{ab} \wedge R^{ab}, \qquad (R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb})$$

  while the translation group is ignored!

2-CATEGORIES AND 2-GROUPS

Category theory

- a category is a structure with "objects" and "morphisms",
- a group is a category with only one object and invertible morphisms.

2-category theory

- a 2-category is a structure with "objects", "morphisms" and "2-morphisms",
- a 2-group is a category with only one object and invertible morphisms and 2-morphisms.

Crossed module $(G, H, \triangleright, \partial)$

- G and H are Lie groups,
- $\triangleright$ is an action of G on H ($\triangleright : G \times H \to H$),
- ∂ is a homomorphism of H on G ($\partial : H \to G$).

Theorem: every 2-group is isomorphic to an appropriate crossed module

POINCARÉ 2-GROUP

Properties of the Poincaré 2-group:

- $(G, H, \triangleright, \partial)$, where:

  $$G = SO(3,1), \qquad H = \mathbb{R}^4, \qquad \triangleright : SO(3,1) \times \mathbb{R}^4 \to \mathbb{R}^4 \qquad \partial : \mathbb{R}^4 \to SO(3,1)$$

- Lorentz group has a connection 1-form ω, but the 2-Poincaré structure generates in addition a 2-form β, such that (ω, β) is called a 2-connection, and transforms as

  $$\omega \to g^{-1}\omega g + g^{-1}dg, \qquad \beta \to g^{-1} \triangleright \beta, \qquad (g : \mathcal{M}_4 \to SO(3,1))$$

  $$\omega \to \omega + \underbrace{\partial\eta}_{0}, \qquad \beta \to \beta + d\eta + \omega \wedge^{\triangleright} \eta + \underbrace{\eta \wedge \eta}_{0}, \qquad (\eta : \mathcal{M}_4 \to \mathbb{R}^4)$$

- one can introduce line holonomies and surface holonomies

  $$g_l(\omega) = \exp \int_l \omega, \qquad h_f(\beta) = \exp \int_f \beta,$$

- one can associate the $BFCG$ (also called $2BF$) action to the Poincaré 2-group:

  $$S = \int B_{ab} \wedge R^{ab} + C_a \wedge G^a, \qquad (G^a = d\beta^a + \omega^a{}_b \wedge \beta^b).$$

TETRAD FIELDS IN THE $BFCG$ ACTION

Note that the Lagrange multiplier C^a

- is a 1-form,
- transforms as

$$C \to g^{-1} \triangleright C, \qquad C \to C \quad \text{wrt. } \eta \text{ transformations},$$

- has an equation of motion $\nabla C^a = 0$.

The multiplier C has exactly the same properties as the tetrad e!

Therefore,

$$\left.\begin{array}{ll} \bullet\ \textbf{identify:} & C^a \equiv e^a, \\ \bullet\ \textbf{rename:} & BFCG \to BFEG, \end{array}\right\} \boxed{\textbf{KEY STEP}}$$

and rewrite the action as

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a .$$

This action is topological, and the 2-group structure enables us to perform the spinfoam-like quantization.

NEW ACTION FOR GENERAL RELATIVITY

The $BFCG$ action can be constrained to give GR:

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}} .$$

Equations of motion are:

- $\delta\phi :$ $\qquad B^{ab} - \varepsilon^{abcd} e_c \wedge e_d = 0,$
- $\delta\beta :$ $\qquad \nabla e^a = 0,$
- $\delta\omega :$ $\qquad \nabla B^{ab} - e^{[a} \wedge \beta^{b]} = 0,$
- $\delta B :$ $\qquad R^{ab} - \phi^{ab} = 0,$
- $\delta e :$ $\qquad \nabla \beta_a + 2\varepsilon_{abcd} \phi^{bc} \wedge e^d = 0,$

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Equations of motion are (after some cleaning-up. . .):

- equations that determine the multipliers and β:

$$\phi^{ab} = R^{ab}, \qquad B^{ab} = \varepsilon^{abcd} e_c \wedge e_d, \qquad \beta^a = 0$$

- Einstein equations:

$$\varepsilon_{abcd} R^{bc} \wedge e^d = 0,$$

- no-torsion equation:

$$\nabla e^a = 0.$$

This is classically equivalent to general relativity!

THE SPINCUBE MODEL

The spincube quantization procedure:

(1) rewrite GR action as a topological theory plus constraint:

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}},$$

(2) quantize the $BFCG$ sector by (a) triangulating the spacetime manifold, (b) defining the path integral

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \int \mathcal{D}e \int \mathcal{D}\beta \exp \left[i \sum_{\Delta} B_{\Delta} R_{\Delta} + \sum_{l} e_l G_l \right] = \ldots =$$

$$= \sum_{\Lambda} \prod_{p} A_1(\Lambda_p) \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v),$$

where Λ are irreducible 2-representations of Poincaré 2-group, while A_1, A_2 and A_4 are chosen such that Z is a topological invariant (a 2-TQFT),

THE SPINCUBE MODEL

The spincube quantization procedure:

(3) enforce the constraint $B^{ab} = \varepsilon^{abcd} e_c \wedge e_d$ by projecting representations Λ to a subset that satisfies the Heron formula for the area of a triangle,

$$|m_f| l_p^2 = A(\Delta) \equiv \sqrt{s(s-l_1)(s-l_2)(s-l_3)}, \qquad (s = \frac{l_1 + l_2 + l_3}{2}),$$

(4) redefine the vertex amplitudes A_1, A_2 and A_4 so that the theory is finite and has a correct classical limit

$\Rightarrow$ obtain a non-topological path integral definition of the theory, with local degrees of freedom.

Main achievements:

- geometry is Regge-like at the Planck scale,
- a single semiclassical limit can be established,
- matter coupling is straightforward,
- easier to calculate with.

MATTER FIELDS

Introduction of matter fields is straightforward:

- at the classical level the fermionic matter can be added to the action due to the explicit presence of the tetrads in the topological sector:

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a - \phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right) +$$

$$+ i\kappa \int \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \bar{\psi} \left(\gamma^d \overset{\leftrightarrow}{d} + \{\omega, \gamma^d\} + \frac{im}{2}\, e^d \right) \psi -$$

$$- i\frac{3\kappa}{4} \int \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c\, \bar{\psi} \gamma_5 \gamma^d \psi, \qquad (\kappa = \frac{8}{3}\pi l_p).$$

- scalar fields, Yang-Mills fields, Immirzi parameter, cosmological constant, . . . , can be added in a similar manner,

- performing the spincube quantization with this new action amounts to introducing additional labels and changes in the vertex amplitude A_4, in order to describe the matter degrees of freedom and their coupling to gravity.

APPLICATIONS OF SPINCUBE MODEL

How can any of this be useful?

- spincube quantization provides one with a concrete quantum theory of gravity with matter,
- one can calculate the effective action using the discretization of the QFT formula

$$e^{i\Gamma[\phi]} = \int \mathcal{D}\varphi \exp\left[iS[\phi+\varphi] - i\int d^4x \frac{\delta\Gamma[\phi]}{\delta\phi}\varphi \right],$$

When matter fields are present in the model, quantum corrections in the effective action can enable one to discuss:

- resolution of the black hole and cosmological singularities,
- detailed analysis of the black hole information paradox,
- renormalization properties of QFT,
- deep Planck-scale regime of space, time and matter,
- motivation for further fundamental questions...

THANK YOU!