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A double-well SUSY matrix model for 2D type IIA superstrings in RR background

Fumihiko Sugino
(Okayama Inst. Quantum Phys.)

Based on collaboration with T. Kuroki

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1 Introduction

◇ Solvable matrix models for 2D quantum gravity or noncritical string theories were vigorously investigated around 1990.

- as toy models for critical string theories, in particular focused on nonperturbative aspects.

- But, little has been known about (solvable) matrix models corresponding to noncritical superstrings with target-space SUSY.
  We would like to consider such matrix models.

- We hope our analysis helpful to analyze Yang-Mills type matrix models for critical strings.

◇ We previously considered a simple SUSY matrix model: [Kuroki-F.S. 2009]

$$S_{\rm MM} = N{\rm tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right],$$

where

$$\left.\begin{array}{ll} B, \phi & : \text{Bosonic} \\ \psi, \bar{\psi} & : \text{Fermionic} \end{array}\right\} N \times N \text{ hermitian matrices.}$$

- SUSY:

$$\begin{array}{llll} Q\phi = \psi, & Q\psi = 0, & Q\bar{\psi} = -iB, & QB = 0, \\ \bar{Q}\phi = -\bar{\psi}, & \bar{Q}\bar{\psi} = 0, & \bar{Q}\psi = -iB, & \bar{Q}B = 0. \end{array}$$

$\Rightarrow Q^2 = \bar{Q}^2 = 0$ (nilpotent)

- B, ψ, $\bar{\psi}$ integrated out

$$S_{\rm MM} \to N{\rm tr}\,\frac{1}{2}(\phi^2 - \mu^2)^2 - \ln\det(\phi \otimes 1\!\!1_N + 1\!\!1_N \otimes \phi)$$

↑

Double-well scalar potential

◇ Large-N saddle point equation for $\rho(x) \equiv \frac{1}{N}\mathrm{tr}\,\delta(x-\phi)$:

$$\int dy\,\rho(y)\,\mathrm{P}\frac{1}{x-y} + \int dy\,\rho(y)\,\mathrm{P}\frac{1}{x+y} = x^3 - \mu^2 x$$

SUSY preserving large-N solution with filling fraction (ν_+, ν_-):

$(\nu_+ + \nu_- = 1)$ [Kuroki-F.S. 2009]

$$\rho(x) = \begin{cases} \frac{\nu_+}{\pi}\, x\sqrt{(x^2-a^2)(b^2-x^2)} & (a < x < b) \\ \frac{\nu_-}{\pi}\, |x|\sqrt{(x^2-a^2)(b^2-x^2)} & (-b < x < -a) \end{cases}$$

with $a = \sqrt{\mu^2 - 2}$, $b = \sqrt{\mu^2 + 2}$.

- Exists for $\mu^2 > 2$.
  (SUSY breaking one-cut solution for $\mu^2 < 2$. [Kuroki-F.S. 2010])
- a and b are independent of $\nu_\pm$! $\Leftarrow$ Characteristic of SUSY model

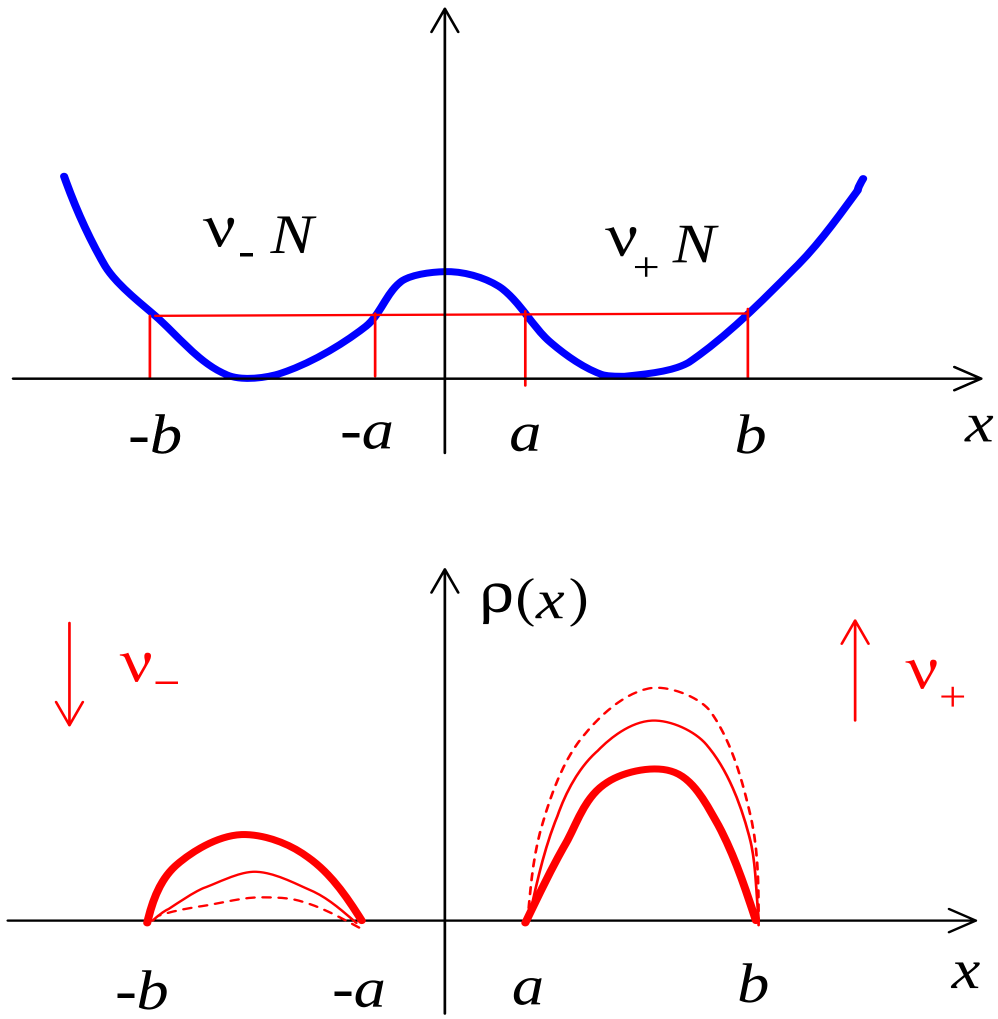

Figure 1: Double-well scalar potential (upper panel) and the eigenvalue distribution for $\nu_+ > \nu_-$ (lower panel).

Large-N saddle point equation:

$$\int dy\, \rho(y)\, \mathrm{P}\frac{1}{x-y} + \int dy\, \rho(y)\, \mathrm{P}\frac{1}{x+y} = x^3 - \mu^2 x$$

$\uparrow$

Effect from fermions

- LHS

$\Rightarrow$ An eigenvalue at $x \in [-b, -a] \cup [a, b]$ feels repulsive force from the other eigenvalues as well as from their mirror images.

$\Rightarrow$ The force independent of $\nu_\pm$!

(Characteristic of SUSY, No analog in bosonic double-well matrix models)

- (large-N free energy) $= 0$, $\quad \langle \frac{1}{N} \mathrm{tr}\, B^n \rangle = 0 \ (n = 1, 2, \cdots)$
  strongly suggest that SUSY is preserved.

Note that $\mathrm{tr}\, B^n = Q\, \mathrm{tr}\, (i\bar{\psi} B^{n-1}) = \bar{Q}\, \mathrm{tr}(i\psi B^{n-1})$.

$\Rightarrow$ The SUSY minima are infinitely degenerate, parametrized by (ν_+, ν_-).

◇ In this talk,

- we compute correlation functions of this matrix model (in sections 2–5).
  ($\longrightarrow$ Logarithmic critical behavior)

- We discuss correspondence between the matrix model and 2D type IIA superstring theory on an nontrivial RR background (in sections 6 & 7).

◇ The logarithmic critical behavior is somewhat reminiscent of the $c = 1$ matrix model (matrix quantum mechanics) [Kazakov-Migdal 1988] or the Penner model (zero-dimensional matrix model). [Distler-Vafa 1991]

$$Z_{\mathrm{Penner}} = \int d^{N^2}M \ \exp[N t \,\mathrm{tr}\{M + \ln(1-M)\}]$$

$\Rightarrow$ Our matrix model $\sim$ a SUSY version of the Penner model
$\sim$ 2D superstring with target-space SUSY.

Note:

- This matrix model is equivalent to the $O(n=-2)$ model on a random surface [Kostov 1989]:

$$Z_{O(n)} = \int d^{N^2}\phi \, e^{-N\mathrm{tr}\, V(\phi)} \, \det(\phi \otimes \mathbb{1}_N + \mathbb{1}_N \otimes \phi)^{-n/2}$$

with $V(\phi) = \frac{1}{2}(\phi^2 - \mu^2)^2$.

- Its critical behavior is described by $c = -2$ topological gravity (i.e. Gaussian one-matrix model). [Kostov-Staudacher 1992]

- It is easily seen by the Nicolai mapping $H = \phi^2$.

[Gaiotto-Rastelli-Takayanagi 2004]

Partition function in the (ν_+, ν_-) sector becomes

$$Z_{\mathrm{MM}}^{(\nu_+,\nu_-)} \Rightarrow (-1)^{\nu_- N} \int d^{N^2}H \, e^{N\mathrm{tr}\, \frac{1}{2}(H-\mu^2)^2}.$$

But, the H-integration is over positive definite hermitian matrices.

For $\mathrm{tr}\, \phi^{2n}$ or $\mathrm{tr}\, B^n$, this approach is applicable at least in $\frac{1}{N}$-expansion.

However, $\mathbf{tr}\,\boldsymbol{\phi}^{\mathbf{2}\boldsymbol{n}+\mathbf{1}}$, $\mathbf{tr}\,\boldsymbol{\psi}^{\mathbf{2}\boldsymbol{n}+\mathbf{1}}$, $\mathbf{tr}\,\bar{\boldsymbol{\psi}}^{\mathbf{2}\boldsymbol{n}+\mathbf{1}}$, ... are not observables in the topological gravity.

- $\mathbf{tr}\,\boldsymbol{\phi}^{\mathbf{2}\boldsymbol{n}+\mathbf{1}} = \mathbf{tr}\,\boldsymbol{H}^{\boldsymbol{n}+\frac{1}{2}}$ is singular at the origin.
- Note that $\mathbf{tr}\,\boldsymbol{\psi}^{\mathbf{2}\boldsymbol{n}} = \mathbf{tr}\,\bar{\boldsymbol{\psi}}^{\mathbf{2}\boldsymbol{n}} = \mathbf{0}$.

Actually, we see nontrivial logarithmic critical behavior for these operators.

Curious observation:

◇ Suppose that ψ and $\bar{\psi}$ correspond to target-space fermions in the corresponding superstring theory.

$$\psi \Leftrightarrow \text{(NS,R) sector}, \qquad \bar{\psi} \Leftrightarrow \text{(R,NS) sector}.$$

Then,

$$\begin{aligned}(-1)^{\mathrm{F}_L} &: \quad \psi \to \psi, \qquad \bar{\psi} \to -\bar{\psi}, \\ (-1)^{\mathrm{F}_R} &: \quad \psi \to -\psi, \qquad \bar{\psi} \to \bar{\psi}.\end{aligned}$$

In order for the matrix model action to be invariant under $(-1)^{\mathrm{F}_L}$ and $(-1)^{\mathrm{F}_R}$,

$$\begin{aligned}(-1)^{\mathrm{F}_L} &: \quad B \to B, \qquad \phi \to -\phi, \\ (-1)^{\mathrm{F}_R} &: \quad B \to B, \qquad \phi \to -\phi.\end{aligned}$$

Recall $S_{\mathrm{MM}} = N\,\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}\{\phi, \psi\}\right]$.

This means

$$B \Leftrightarrow \text{(NS,NS) sector}, \qquad \phi \Leftrightarrow \text{(R,R) sector}.$$

2 Planar one-point functions

$$\left\langle \frac{1}{N} \operatorname{tr} \phi^n \right\rangle_0 = \int dx\, x^n \rho(x)$$
$$= (\nu_+ + (-1)^n \nu_-) (2+\mu^2)^{n/2} F\left(-\frac{n}{2}, \frac{3}{2}, 3; \frac{4}{2+\mu^2}\right)$$

- reduces to a polynomial of μ^2 for n even:
$$\left\langle \frac{1}{N} \operatorname{tr} \phi^2 \right\rangle_0 = \mu^2, \qquad \left\langle \frac{1}{N} \operatorname{tr} \phi^4 \right\rangle_0 = 1 + \mu^4, \qquad \cdots.$$
($c = -2$ topological gravity)

- exhibits logarithmic singular behavior as $\mu^2 \to 2$ for n odd:

$$\omega \equiv \tfrac{1}{4}(\mu^2 - 2)$$

$$\left\langle \frac{1}{N} \operatorname{tr} \phi^{2k+1} \right\rangle_0 = (\nu_+ - \nu_-) \left[(\text{const.})\, \omega^{k+2} \ln \omega + (\text{less singluar}) \right].$$

Explicit form for a first few expectation values:

$$\left\langle \frac{1}{N} \mathrm{tr}\, \phi \right\rangle_0 = (\nu_+ - \nu_-) \left[\frac{64}{15\pi} + \frac{16}{3\pi}\omega + \frac{2}{\pi}\omega^2 \ln \omega + \mathcal{O}(\omega^2) \right],$$

$$\left\langle \frac{1}{N} \mathrm{tr}\, \phi^3 \right\rangle_0 = (\nu_+ - \nu_-) \left[\frac{1024}{105\pi} + \frac{128}{5\pi}\omega + \frac{16}{\pi}\omega^2 + \frac{4}{\pi}\omega^3 \ln \omega + \mathcal{O}(\omega^3) \right],$$

$$\left\langle \frac{1}{N} \mathrm{tr}\, \phi^5 \right\rangle_0 = (\nu_+ - \nu_-) \left[\frac{8192}{315\pi} + \frac{2048}{21\pi}\omega + \frac{128}{\pi}\omega^2 + \frac{160}{3\pi}\omega^3 + \frac{10}{\pi}\omega^4 \ln \omega + \mathcal{O}(\omega^4) \right].$$

$$F\left(-\tfrac{n}{2}, \tfrac{3}{2}, 3; \tfrac{1}{1+\omega}\right) = F_{\mathrm{non}} + F_{\mathrm{univ}} \qquad \text{for } n \text{ odd.}$$

↗ Nonuniversal part ("lattice artifact")

↖ Universal part (relevant to "continuum physics")

3 Planar two-point functions (Bosons)

It turns out that they take a quadratic form of F.

- "Even-even" correlators:

$$\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k}\,\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\right\rangle_{C,0} = (\text{polynomial of } \omega \text{ indep. of } (\nu_+ - \nu_-))$$

($c = -2$ topological gravity)

- "Even-odd" correlators:

$$\left\langle \Phi_{2k+1}\,\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\right\rangle_{C,0} = (\nu_+ - \nu_-)(\text{const.})\,\omega^{k+1}\ln\omega + (\text{less singular})$$

- "Odd-odd" correlators:

$$\langle \Phi_{2k+1}\,\Phi_{2\ell+1}\rangle_{C,0} = (\nu_+ - \nu_-)^2(\text{const.})\,\omega^{k+\ell+1}(\ln\omega)^2 + (\text{less singular}),$$

where in order to subtract nonuniversal contribution we took a basis of the "odd" operators (operator mixing):

$$\Phi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} + (\nu_+ - \nu_-)\sum_{i=1}^{k}\alpha_{2k+1,2i}(\omega)\frac{1}{N}\mathrm{tr}\,\phi^{2i}$$

$$\begin{array}{ccc} \updownarrow & \updownarrow & \updownarrow \\ F_{\mathrm{univ}} & F\left(\cdot,\cdot,\cdot;\frac{1}{1+\omega}\right) & -F_{\mathrm{non}} \end{array}$$

with $\alpha_{2k+1,2i}(\omega)$ being a regular function at $\omega = 0$.

- From the previous observation ($\phi \Leftrightarrow$ (R, R) sector), $(\nu_+ - \nu_-) \Leftrightarrow$ RR charge

- Φ_{2k+1} has RR charge.

4 Planar three-point functions (Bosons)

Cubic form of $\boldsymbol{F}$.

We obtain

$$\langle \Phi_1 \Phi_1 \Phi_1 \rangle_{C,0} = (\nu_+ - \nu_-)^3 \left[\frac{1}{16\pi^3} (\ln \omega)^3 + \mathcal{O}((\ln \omega)^2) \right],$$

$$\langle \Phi_1 \Phi_1 \Phi_3 \rangle_{C,0} = (\nu_+ - \nu_-)^3 \left[\frac{2}{\pi^3} + \frac{3}{8\pi^3} \omega (\ln \omega)^3 + \mathcal{O}(\omega (\ln \omega)^2) \right].$$

4 Planar three-point functions (Bosons)

Cubic form of F.

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- The results so far suggest

$$\langle \Phi_{2k_1+1} \cdots \Phi_{2k_n+1} \rangle_{C,0} = (\nu_+ - \nu_-)^n (\text{const.})\, \omega^{2-\gamma+\sum_{i=1}^n (k_i - 1)} (\ln \omega)^n + (\text{less singular})$$

with $\gamma = -1$. ← string susceptibility of $c = -2$ topological gravity

Gravitational scaling dimension of Φ_{2k+1} is k, besides the logarithmic factors $(\ln \omega)^n$.

5 Planar two-point functions (Fermions)

For fermions, we obtain

$$\left\langle \Psi_{2k+1} \bar{\Psi}_{2\ell+1} \right\rangle_{C,0} = \delta_{k,\ell} \,(\text{const.})\, (\nu_+ - \nu_-)^{2k+1} \omega^{2k+1} \ln \omega \\ +(\text{less singular})$$

with

$$\Psi_{2k+1} = \frac{1}{N} \mathrm{tr}\, \psi^{2k+1} + (\text{mixing}),$$
$$\bar{\Psi}_{2k+1} = \frac{1}{N} \mathrm{tr}\, \bar{\psi}^{2k+1} + (\text{mixing}).$$

$\Rightarrow$ Ψ_{2k+1} and $\bar{\Psi}_{2k+1}$ have the dimension k same as Φ_{2k+1}, besides the logarithmic factor.

6 2D type IIA superstring [Kutasov-Seiberg 1990, Ita-Nieder-Oz 2005]

- (Target space) $= (x, \varphi)$,
  where $x \in S^1$ with self-dual radius $(R = 1)$ and φ: Liouville.
  (↖Same as the Penner model!)

- Holomorphic EM tensor (except ghost part) on string world-sheet:

$$T_m = -\frac{1}{2}(\partial x)^2 - \frac{1}{2}\psi_x \partial \psi_x - \frac{1}{2}(\partial \varphi)^2 + \frac{Q}{2}\partial^2 \varphi - \frac{1}{2}\psi_\ell \partial \psi_\ell$$

  with $Q = 2$.

- Target-space SUSY is nilpotent.

$$q_+(z) = e^{-\frac{1}{2}\phi - \frac{i}{2}H - ix(z)}, \qquad Q_+ = \oint \frac{dz}{2\pi i}\, q_+(z),$$

$$\bar{q}_-(\bar{z}) = e^{-\frac{1}{2}\bar{\phi} + \frac{i}{2}\bar{H} + i\bar{x}}(\bar{z}), \qquad \bar{Q}_- = \oint \frac{d\bar{z}}{2\pi i}\, \bar{q}_-(\bar{z}),$$

  where $\psi_\ell \pm i\psi_x = \sqrt{2} e^{\mp iH}$.

  $\Rightarrow Q_+^2 = \bar{Q}_-^2 = 0.$ (← Same as the matrix model!)

- Vertex operators (holomorphic sector):

$$\text{NS sector } (-1)\text{-picture}: \quad T_k(z) = e^{-\phi+ikx+p_\ell\varphi}(z)$$

$$\text{R sector } (-\tfrac{1}{2})\text{-picture}: \quad V_{k,\,\epsilon}(z) = e^{-\frac{1}{2}\phi+\frac{i}{2}\epsilon H+ikx+p_\ell\varphi}(z)$$

with $\epsilon = \pm 1$.

Locality with supercurrents, mutual locality, superconformal inv., level matching

$\Rightarrow$ physical vertex operators

$p_\ell = 1 - |k|$ ($\leftarrow$ conformal inv. & Seiberg's locality bound)

$k = \epsilon|k|$ ($\leftarrow$ Dirac equation constraint)

Note

The branch of $p_\ell = 1 + |k|$ does not satisfy Seiberg's locality bound ($p_\ell < 1$).

$\Rightarrow$ Insertion of such "nonlocal" vertex operators cannot be regarded as a local disturbance on string world-sheet.

Corresponding wave function (disk with the vertex op. inserted) peaks at φ large:

$$\Psi \sim \frac{1}{g_{st}} \,(\text{Vertex op.}) \sim e^{-\frac{Q}{2}\varphi}\, e^{p_\ell \varphi} = e^{(p_\ell - 1)\varphi}.$$

Dynamical metric on string world-sheet : $g_{ab} = \hat{g}_{ab}\, e^{\frac{2}{Q}\varphi}$

- $\varphi \sim +\infty$: large geometry (nonlocal, macroscopic)
- $\varphi \sim -\infty$: small geometry (local, microscopic)

Winding background: [Ita-Nieder-Oz 2005]

(NS, NS) : $T_k(z)\,\bar{T}_{-k}(\bar{z})$ $(k \in \mathbf{Z} + \frac{1}{2})$ "tachyon" winding

(R+, R−) : $V_{k,+1}(z)\,\bar{V}_{-k,-1}(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$

(R−, R+) : $V_{-k,-1}(z)\,\bar{V}_{k,+1}(\bar{z})$ $(k = 0, 1, 2, \cdots)$

RR 2-form field strength winding

(NS, R−) : $T_{-k}(z)\,\bar{V}_{-k,-1}(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$ fermion(−) momentum

(R+, NS) : $V_{k,+1}(z)\,\bar{T}_k(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$ fermion(+) momentum

Interesting observation:
Let us assume the correspondence of supercharges between the matrix model and the type IIA theory:

$$(Q, \bar{Q}) \Leftrightarrow (Q_+, \bar{Q}_-).$$

$\Rightarrow$ SUSY transformation properties & the observation before lead to

$$\Phi_1 = \frac{1}{N}\mathrm{tr}\,\phi \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\, \bar{V}_{-\frac{1}{2},-1}(\bar{z}) \qquad \text{(R+, R−)},$$

$$\Psi_1 = \frac{1}{N}\mathrm{tr}\,\psi \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\, \bar{V}_{-\frac{1}{2},-1}(\bar{z}) \qquad \text{(NS, R−)},$$

$$\bar{\Psi}_1 = \frac{1}{N}\mathrm{tr}\,\bar{\psi} \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\, \bar{T}_{\frac{1}{2}}(\bar{z}) \qquad \text{(R+, NS)},$$

$$\frac{1}{N}\mathrm{tr}(-iB) \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\, \bar{T}_{\frac{1}{2}}(\bar{z}) \qquad \text{(NS, NS)}.$$

Quartet w.r.t. $(Q, \bar{Q})$ $\Leftrightarrow$ Quartet w.r.t. $(Q_+, \bar{Q}_-)$

Furthermore, it is natural to extend it to higher $k(=1,2,\cdots)$ as

$$\Phi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} + (\text{mixing}) \Leftrightarrow \int d^2z\, V_{k+\frac{1}{2},+1}(z)\,\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}),$$

$$\Psi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\psi^{2k+1} + (\text{mixing}) \Leftrightarrow \int d^2z\, T_{-k-\frac{1}{2}}(z)\,\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}),$$

$$\bar{\Psi}_{2k+1} = \frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2k+1} + (\text{mixing}) \Leftrightarrow \int d^2z\, V_{k+\frac{1}{2},+1}(z)\,\bar{T}_{k+\frac{1}{2}}(\bar{z}),$$

$$\frac{1}{N}\mathrm{tr}(-iB)^{k+1} + (\text{mixing}) \Leftrightarrow \int d^2z\, T_{-k-\frac{1}{2}}(z)\,\bar{T}_{k+\frac{1}{2}}(\bar{z}).$$

(Single trace operators in the matrix model) $\Leftrightarrow$ (Integrated vertex operators in IIA)

(Powers of matrices) $\Leftrightarrow$ (Windings or Momenta)

Note:

- RR 2-form field strength in (R$-$, R$+$) is a singlet under the target-space SUSYs Q_+, $\bar{Q}_-$, and appears to have no matrix-model counterpart.
- Expectation values of operators with nonzero Ramond charge (e.g. $\langle\Phi_{2k+1}\rangle_0$) are nonvanishing in the matrix model.

$\Rightarrow$ The matrix model is considered to correspond to IIA on a background of the RR 2-form.

Let us check the correspondence by computing amplitudes in IIA theory.

7 Correspondence between the matrix model and the IIA theory

◇ Correlation functions among integrated vertex operators in IIA on the trivial background:

$$\left\langle \prod_i \mathcal{V}_i \right\rangle = \frac{1}{\text{Vol.(CKV)}} \int \mathcal{D}(x, \varphi, H, \text{ghosts})\, e^{-S_{\rm CFT}} e^{-S_{\rm int}} \prod_i \mathcal{V}_i,$$

$$S_{\rm CFT} = \frac{1}{2\pi} \int d^2 z \left[\partial x \bar{\partial} x + \partial \varphi \bar{\partial} \varphi + \frac{Q}{4} \sqrt{\hat{g}} \hat{R} \varphi + \partial H \bar{\partial} H + (\text{ghosts}) \right],$$

$$S_{\rm int} = \omega \int d^2 z\, T^{(0)}_{-\frac{1}{2}}(z) \bar{T}^{(0)}_{\frac{1}{2}}(\bar{z}) \qquad (\leftarrow \text{0-picture (NS, NS) "tachyon"})$$

7 Correspondence between the matrix model and the IIA theory

◇ Correlation functions among integrated vertex operators in IIA on the trivial background:

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$$S_{\rm CFT} = \frac{1}{2\pi} \int d^2z \left[\partial x \bar{\partial} x + \partial\varphi \bar{\partial}\varphi + \frac{Q}{4}\sqrt{\hat{g}}\hat{R}\varphi + \partial H \bar{\partial} H + (\text{ghosts}) \right],$$

$$S_{\rm int} = \omega \int d^2z\, T^{(0)}_{-\frac{1}{2}}(z) \bar{T}^{(0)}_{\frac{1}{2}}(\bar{z}) \qquad (\leftarrow \text{0-picture (NS, NS) “tachyon”})$$

◇ Correlation functions in IIA on (R−, R+) background:

$$\left\langle\!\left\langle \prod_i \mathcal{V}_i \right\rangle\!\right\rangle \equiv \left\langle \left(\prod_i \mathcal{V}_i \right) e^{W_{\rm RR}} \right\rangle,$$

where $W_{\rm RR}$ is invariant under the target-space SUSYs:

$$W_{\rm RR} = (\nu_+ - \nu_-) \sum_{k\in Z} a_k\, \omega^{k+1} \mathcal{V}_k^{\rm RR}, \qquad (a_k : \text{numerical consts.})$$

$$\mathcal{V}_k^{\rm RR} \equiv \begin{cases} \int d^2z\, V_{k,\,-1}(z) \bar{V}_{-k,\,+1}(\bar{z}) & (p_\ell = 1 - |k|,\ k = 0, -1, -2, \cdots) \\ \int d^2z\, V_{-k,\,-1}^{(\rm nonlocal)}(z) \bar{V}_{k,\,+1}^{(\rm nonlocal)}(\bar{z}) & (p_\ell = 1 + |k|,\ k = 1, 2, \cdots). \end{cases}$$

Note

- We treat the RR background for $(\nu_+ - \nu_-)$ small as exponentiated vertex operators:

$$\left\langle\!\!\left\langle \prod_i \mathcal{V}_i \right\rangle\!\!\right\rangle \equiv \left\langle \left(\prod_i \mathcal{V}_i\right) e^{W_{\rm RR}} \right\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left\langle \left(\prod_i \mathcal{V}_i\right) (W_{\rm RR})^n \right\rangle.$$

- Liouville-like interaction

$$S_{\rm int} = \omega \int d^2z\, T^{(0)}_{-\frac{1}{2}}(z) \bar{T}^{(0)}_{\frac{1}{2}}(\bar{z}) \quad \Leftrightarrow \quad N(\mu^2 - 2){\rm tr}(-iB) \in S_{\rm MM}$$

◇ Standard Liouville theory computation for amplitudes leads to:

- $$\left\langle \frac{1}{N}\mathrm{tr}(-iB)\,\Phi_{2k+1}\right\rangle_0 = -\frac{1}{4}\partial_\omega \left\langle \Phi_{2k+1}\right\rangle_0 \sim (\nu_+ - \nu_-)\,\omega^{k+1}\ln\omega \Leftrightarrow$$
$$-\frac{1}{4}(\nu_+ - \nu_-)\sum_{\ell\in\mathbf{Z}} a_\ell\,\omega^{\ell+1}\left\langle \left(\int T_{-\frac{1}{2}}\bar{T}_{\frac{1}{2}}\right)\left(\int V_{k+\frac{1}{2},+1}\bar{V}_{-k-\frac{1}{2},-1}\right)\mathcal{V}^{\mathrm{RR}}_\ell\right\rangle$$
$$= -\frac{1}{2}(\nu_+ - \nu_-)\,a_k\,\omega^{k+1}\ln\omega,$$

- $$\left\langle \Phi_{2k_1+1}\Phi_{2k_2+1}\right\rangle_{C,0} \sim (\nu_+ - \nu_-)^2\,\omega^{k_1+k_2+1}(\ln\omega)^2 \Leftrightarrow$$
$$\frac{1}{2}(\nu_+ - \nu_-)^2\sum_{\ell_1,\ell_2\in\mathbf{Z}} a_{\ell_1}a_{\ell_2}\,\omega^{\ell_1+\ell_2+2}$$
$$\times\left\langle \left(\int V_{k_1+\frac{1}{2},+1}\bar{V}_{-k_1-\frac{1}{2},-1}\right)\left(\int V_{k_2+\frac{1}{2},+1}\bar{V}_{-k_2-\frac{1}{2},-1}\right)\mathcal{V}^{\mathrm{RR}}_{\ell_1}\mathcal{V}^{\mathrm{RR}}_{\ell_2}\right\rangle$$
$$= (\nu_+ - \nu_-)^2\,2\pi\,a_{k_1+k_2}\,a_{-1}\left(\frac{(k_1+k_2)!}{k_1!k_2!}\right)^2\omega^{k_1+k_2+1}(\ln\omega)^2,$$

$\cdots$

with appropriate regularization by the Liouville volume $V_L = -2\ln\omega$.

- Computation in the type IIA side reproduces the $(\nu_+ - \nu_-)$-dependence and the ω-dependence in the matrix model result!

- Higher powers of $\ln \omega$ comes from resonances to the (R−,R+) background.

8 Summary and discussions

◇ We computed correlation functions in the double-well SUSY matrix model, and discussed its correspondence to 2D type IIA superstring theory on (R−,R+) background by computing amplitudes in both sides.

This is an interesting example of matrix models for superstrings with target-space SUSY, in which various amplitudes are explicitly calculable.

◇ Matrix-model counterpart of positive-winding "tachyons" $\boldsymbol{T_{k-\frac{1}{2}}\bar{T}_{-k+\frac{1}{2}}}$ $(\boldsymbol{k=1,2,\cdots})$?
Similar to the Kontsevich-Penner model (introducing an external matrix source)?
[Imbimbo-Mukhi 1995]

◇ Higher genus amplitudes?

◇ D-brane interpretation of the matrix model?

◇ Case of $(\boldsymbol{\nu}_+ - \boldsymbol{\nu}_-)$ not small?

Related to black-hole (cigar) target space? cf. [Hori-Kapustin 2001]

Thank you very much for your attention!

A The Penner model [Distler-Vafa 1991]

- Partition function

$$Z = \mathcal{N}_P \int d^{N^2}M \; \exp[Nt\,\mathrm{tr}\{M + \ln(1-M)\}]$$

$$= \mathcal{N}_P \int d^{N^2}M \; \exp\left[-Nt\,\mathrm{tr}\sum_{k=2}^{\infty}\frac{1}{k}M^k\right],$$

where $\frac{1}{\mathcal{N}_P} = \int d^{N^2}M \; \exp\left[-Nt\,\mathrm{tr}\,\frac{1}{2}M^2\right]$.

- Free energy

$$\ln Z = \sum_{g=0}^{\infty} N^{2-2g}\mathcal{F}_g,$$

$$\mathcal{F}_g = \frac{B_{2g}}{2g(2g-2)}t^{2-2g}\left((1+t)^{2-2g}-1\right) \qquad \text{for} \qquad g \geq 2$$

$\Rightarrow$ Double scaling limit: $N \to \infty$, $t \to -1$ with $N(1+t) = -\nu$ fixed.

After putting $\nu = -i\mu$, the free energy of $c = 1, R = 1$ string is obtained.

$$\mathcal{F}_g = \frac{|B_{2g}|}{2g(2g-2)}\,\mu^{2-2g} \qquad (g \geq 2)$$

$$|B_{2g}| = (-1)^{g-1} B_{2g}$$

B The Kontsevich-Penner model (W_∞ matrix model)

Extension of the Penner model to include source terms for "tachyon" operators in 2D string (with $\nu \to -\nu$). [Imbimbo-Mukhi 1995]

- Partition function (solution of the W_∞ constraint):

$$Z(t,\bar{t}) = (\det A)^{\nu} \int d^{N^2}M \, \exp\left[\mathrm{tr}\left\{-\nu M A + (\nu - N)\ln M - \nu \sum_{k=1}^{\infty} \bar{t}_k M^k\right\}\right]$$

$$= \int d^{N^2}M \, \exp\left[\mathrm{tr}\left\{-\nu M + (\nu - N)\ln M - \nu \sum_{k=1}^{\infty} \bar{t}_k (M A^{-1})^k\right\}\right].$$

- $\bar{t}_k$ is a source for "tachyons" of negative momentum $-k$ ($\sim \mathrm{tr}\, M^k$).

- A: external $N \times N$ matrix
  Source for positive-momentum "tachyons" t_k is given by the

Kontsevich-Miwa transformation of A:

$$t_k = \frac{1}{\nu k} \mathrm{tr}\, A^{-k}.$$

$\Rightarrow$ Asymmetric treatment for positive/negative-momentum “tachyons”

- “Tachyon” amplitude

$$\langle \mathcal{T}_{k_1} \cdots \mathcal{T}_{k_n} \mathcal{T}_{-l_1} \cdots \mathcal{T}_{-l_m} \rangle = \frac{\partial}{\partial t_{k_1}} \cdots \frac{\partial}{\partial t_{k_n}} \frac{\partial}{\partial \bar{t}_{l_1}} \cdots \frac{\partial}{\partial \bar{t}_{l_m}} \ln Z(t, \bar{t}) \Big|_{t=\bar{t}=0}$$