“Massive” Modification of standard Perturbative QCD with Application to DIS*

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on Analytic Perturbation Theory = APT

"Analytic Perturb Theory" in QCD, the closed theor. scheme [Solovtsov+DVSh-97] without Landau-pole and additional parameters. It stems from imperative of Q^2-analyticity and compatibility with linear integral transformations (like, Fourier one). Incorporates e^{-1/α_s} (algebraic in Q^2) structures. Instead of power PT set $\bar{\alpha}_s(Q^2), \bar{\alpha}_s(Q^2)^2, \ldots \bar{\alpha}_s(Q^2)^k$, one deals with non-power APT expansion set $\{\mathcal{A}_k(Q^2)\}$; $k = 1, 2, \ldots$

with all $\mathcal{A}_k(Q^2)$ regular in the IR.

The first, $\mathcal{A}_1(Q) =$ APT effective coupling,

quantitatively corresponds to results of

Lattice Simulation down to $Q \sim 500$ MeV.

Pleasant Bonus of Recurrent Relations = RecRel

The APT RecRel

$$-\,\frac{1}{k}\,\dot{\mathcal{A}}_k = \beta_0\,\mathcal{A}_{k+1} + \beta_1\,\mathcal{A}_{k+2} + \ldots \qquad (1)$$

looks like a replica of the RG basic diff. eq. and does reduce to it at the UV, as far as $\beta_0^k\,\mathcal{A}_k(Q \to UV) \to (\alpha_s(Q))^k$.

At the same time, RecRel (1) provides one with possibility to change a higher correction for the shift of argument. E.g., at 1-loop case

$$c_1\,\mathcal{A}_1(Q) + c_2\,\mathcal{A}_2(Q) + \cdots \sim c_1\,\mathcal{A}_1(Q - \Delta Q) + o(\mathcal{A}_3)\,,$$

with $\Delta Q \sim e^{c_2/(c_1\,\beta_0)}$. In that follows, we will preserve the RecRel

New "massive" pQCD = MPT

The proposed "massive analytic pQCD" is constructed on the two grounds.

* One is the pQCD itself with one parameter added, the effective "glueball mass", $m_{gl} \lesssim 1\,\mathrm{GeV}$ serving as an IR regulator.

** The second stems out of the ghost-free Analytic Perturbation Theory (= APT) comprising Non-power perturbative expansion that makes it compatible with linear integral transformations.

Motivation for the MPT

This achievement rises the hope for possibility of global fitting down to IR limit. Unhappily, no one of the known ghost-free models is suitable for this goal. The drawback is the **use of UV logs in IR region**.

To approach the global fitting of data (like ones for the BjSR form factor), one needs to have a theoretical framework with (at least) two essential features:

- Correspondence with common pQCD in the UV;
- Correlation with lattice simulation results for the QCD coupling $\alpha_s(Q)$ regular LE behavior.

Outbreak of PT series at low scale, 2

Table 2. Relative contributions of 1-,2-,3- and 4-loop terms

Process		Scale/Gev	*PT terms (in %)*				APT ♡		
the loop number =			1	2	3	4	1	2	3
Bjorken SR	t	1	35	20	**19 !**	**26 ?!**	80	19	1
Bjorken SR	t	1.78	56	21	13	**11 !**	80	19	1
GLS SumRule	t	1.78	58	21	12 **11 !**	75	21	4	
Incl. τ-decay	s	1.78	51	27	14	**7 !**	88	11	1

♡ The 4-loop APT contributions are negligible everywhere.

For illustration : $\alpha_s(1\text{GeV}) \sim 0.55$ and $\alpha_s(1.78\,\text{GeV}) = 0.34\,.$

Motivation for the MPT, 2

Need for theoretical scheme with 2 important features:

- Effective coupling corresponding to QCD coupling $\alpha_s(Q)$ in the UV, and close to lattice-simulated $\alpha_s^{lat}(Q)$ which is regular in the LE region.
- Perturbative expansion over set of non-power functions that provides us with practical convergence (a là Poincaré) and compatibility with linear integral transformations.

To this goal, mass-dependent pQCD modification inspired by our paper [DVSh-99] will be proposed.

MPT = “massive” pQCD

“Gluonic” mass m_{gl} in the MPT

In that paper [hep-ph/9903431], on the base of *massive* Bogoliubov RG, the LE-regular version of pQCD, contained a supposed gluon mass, has been devised.

Below, we construct *massive pQCD* – MPT for short – with one added (but Λ) fitting parameter m_{gl} , which modifies $\alpha_s(Q)$ behavior below 1-2 GeV .

The notion of “gluonic(glueball)” mass ascends to Simonov [90s] and others. In our case, it turns out that quantitatively

$$m_\rho \lesssim m_{gl} \lesssim M_N \, ,$$

m_{gl} lies between ρ-meson and nucleon masses.

MPT = “massive” pQCD

Massive Renorm–group summation*

From 2-loop massive expansion $(z = Q^2/\mu^2, y = m^2/\mu^2)$

$$\alpha_s(z)^{[2]}_{\rm pt} = \alpha_s - \alpha_s^2 A_1(z,y) + \alpha_s^3 \left(A_1^2 - A_2(z,y)\right) + \ldots\,;, \qquad (2)$$

one gets [Blank+DVSh 1956, DVSh-92] RG-invariant "massive" sum

$$\alpha_s(z)^{[2]}_{\rm rg} = \frac{\alpha_s}{1 + \alpha_s\, A_1(z) + \alpha_s \frac{A_2(z)}{A_1(z)}\, \ln(1 + \alpha_s A_1(z))}\,. \qquad (3)$$

In the UV, with Λ parameterization

$$1/\beta_0\, \alpha_s + \ln(Q^2/\mu^2) = \ln(Q^2/\Lambda^2) \equiv \ln x\,. \qquad (4)$$

this yields

$$\alpha_s(Q^2)^{[1]}_{\rm rg} = \frac{1}{\beta_0\, L + \frac{\beta_1}{\beta_0}\ln L}\,; \quad L = \ln\frac{Q^2}{\Lambda^2} \qquad (5)$$

that is "Denominator Representation" of common PDG expression.

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The MPT modelling

The ‘1-loop structure’ in the denominator of (5) we change now for the “long log” with one adjustable parameter

$$\ln x \to L_\xi(x) = \ln(\xi + x)\,, \quad x = \frac{Q^2}{\Lambda^2}\,. \quad (6)$$

At moderate scales the form (at $\phi = 1/\xi\,,\ m_{gl} = \sqrt{\xi}\,\Lambda$)

$$L_\xi(x) = \ln\xi + \ln(1+\phi\,x) = \frac{1}{\beta_0\,\alpha_s} + \ln(1+\phi\,x)\,; \quad \phi\,x = \frac{Q^2}{m_{gl}^2} \quad (7)$$

is more adequate.

Here, transition to the “long log” can be formulated in the mnemonic form

$$\boxed{Q^2 \to Q^2 + m_{gl}^2} \quad (8)$$

MPT = “massive” pQCD

The MPT modelling, 2

At the 2-loop case we use *the same "long log"* $A_2(x\,\phi) = \beta_1 \ln(1+\phi\,x)$ *for the 2-loop contribution.* That is

$$\mathcal{A}^{[2]}_{1,MPT}(x) =$$

$$= \frac{\alpha_s}{1+\alpha_s\,\beta_0 \ln(1+\phi\,x) + \alpha_s \frac{\beta_1}{\beta_0} \ln[1+\alpha_s\,\beta_0 \ln(1+\phi\,x)]}\,. \qquad (9)$$

Note that $\Lambda^{[2loop]}(\xi)$ dependence is rather weak : $\Lambda^{[2]}(5) \sim 335\,\mathrm{MeV}$; $\Lambda^{[2]}(10\pm 2) \sim 315 \mp 10\,\mathrm{MeV}$ with values less than pQCD one $\Lambda^{[3-loops]}_{n_f=3} \sim= 420 \pm 10\,\mathrm{MeV}$.

The MPT modelling, 3

In the MPT devising, we preserve an essential APT feature, the non-polynomial type of "perturbative" expansion; an expansion over a set of functions $\{\mathcal{A}_{k,MPT}(Q^2)\}$ connected by differential recursion relation (with notation for log derivative $\dot{F} = x\,F'$)

$$-\frac{1}{k}\,\dot{\mathcal{A}}_{k,MPT} = \beta_0\,\mathcal{A}_{k+1,MPT}(x) + \beta_1\,\mathcal{A}_{k+2,MPT}(x) + \dots\ , \quad (10)$$

the same as in the APT. Its 1-loop version

$$-\,\frac{1}{k}\,\dot{\mathcal{A}}_{k,MPT} = \beta_0\,\mathcal{A}_{k+1,MPT}(x) + \dots\ , \quad (11)$$

coincides with the 1-loop PT.

Higher MPT expansion functions

This recurrence property ensures the compatibility with linear transformations like transition to the annihilation s-channel and to the distance r-picture.

The "MPT coupling squared". In a particular case $k=1$, this gives the 2nd MPT function

$$\mathcal{A}_{2,MPT}(x) = \left(\mathcal{A}_{1,MPT}\right)^2 \; R(x)\,; \quad R(x) = \frac{\phi\, x}{1+\phi\, x}\,, \tag{12}$$

that turns to zero in the IR limit, like the APT one, but with finite derivative $\mathcal{A}^{\prime}_{2,MPT}(0) = \alpha_s^2\,\phi\,.$

MPT = "massive" pQCD

The third MPT function*

The third MTP expansion function can be estimated by the same eq.(11) with the omitted term with β_1 . This gives

$$2\,\beta_0\,\mathcal{A}^{[1]}_{3,MPT}(x) = -2\,\mathcal{A}_{1,MPT}\,\dot{\mathcal{A}}_{1,MPT}\,R(x) - \mathcal{A}^{2}_{1,MPT}\,\dot{R}(x)\,, \quad (13)$$

that is sufficient for perceiving its IR properties

$$\mathcal{A}^{[1]}_{3,MPT}(0) = 0; \quad \mathcal{A}^{[1]'}_{3,MPT}(0) = -\alpha_s^2\,\phi\,. \quad (14)$$

Now, the MPT "perturbative" expansion similar to (**??**) is

$$\Delta^{[2]}_{MPT}(Q^2) =$$

$$= \tfrac{1}{\pi}\,\mathcal{A}^{[2]}_{1,MPT}(x) + 0.363\,\mathcal{A}^{[1]}_{2,MPT}(x) + 0.652\,\mathcal{A}^{[1]}_{3,MPT}(x)\ldots \quad (15)$$

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Comparison with APT

To compare new model with the APT one, at Fig.1 we give curves for a few values of ξ in the region below 2 GeV for the first 2-loop MPT function (9) vs. the 1st APT (dashed curve).

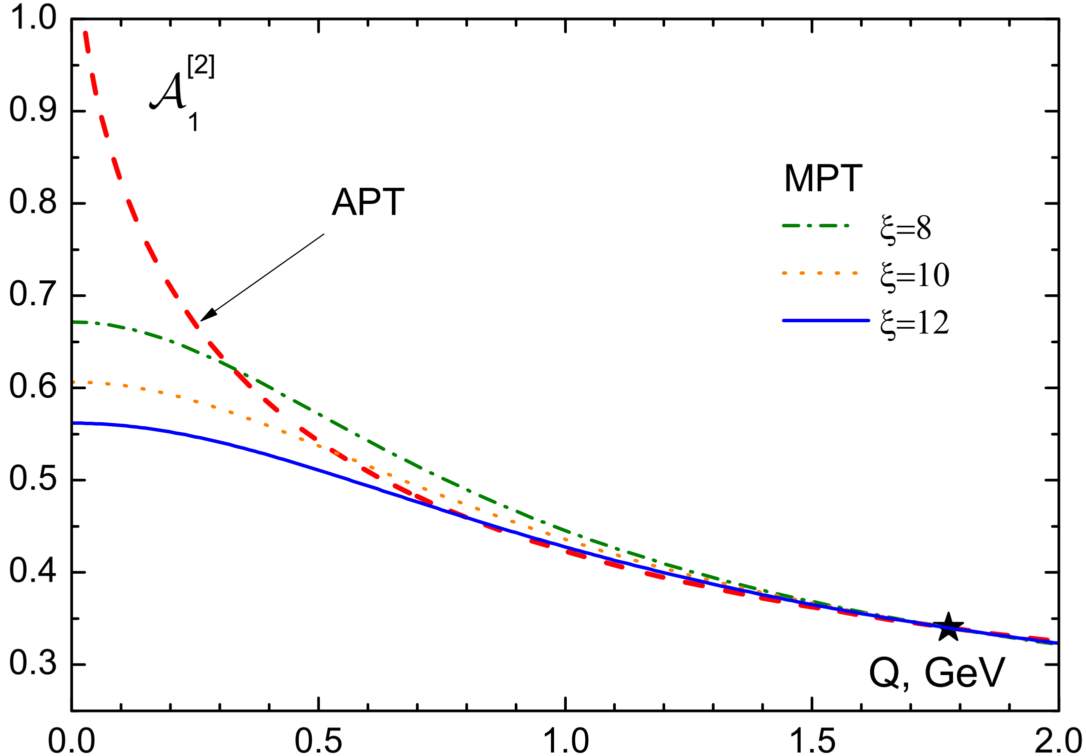


Figure 1: Effective MPT coupling vs. the APT one.

MPT = "massive" pQCD

Comparison with APT, 2

As it follows from the NLO curves, values $\xi = 8 - 10$ seems to be preferable. Indeed, for these values, the first MPT function is reasonably close to first APT one down to 1 GeV . At the same time, around 500 MeV it deviates of APT but just to be more close to the lattice simulations results.

The same set of curves for higher MPT and APT functions

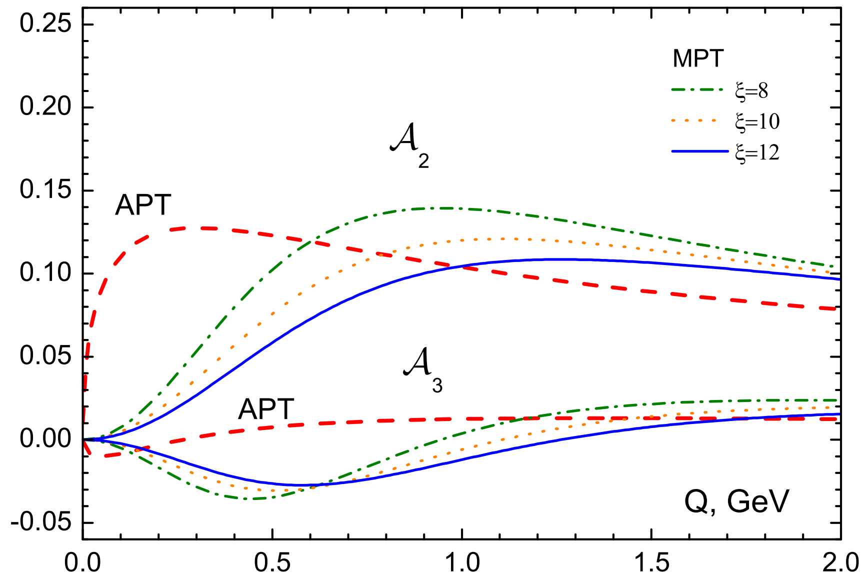


Figure 2: 2nd and 3rd MPT functions vs. the APT ones.

MPT = "massive" pQCD

The "Gluonic" masses in MPT

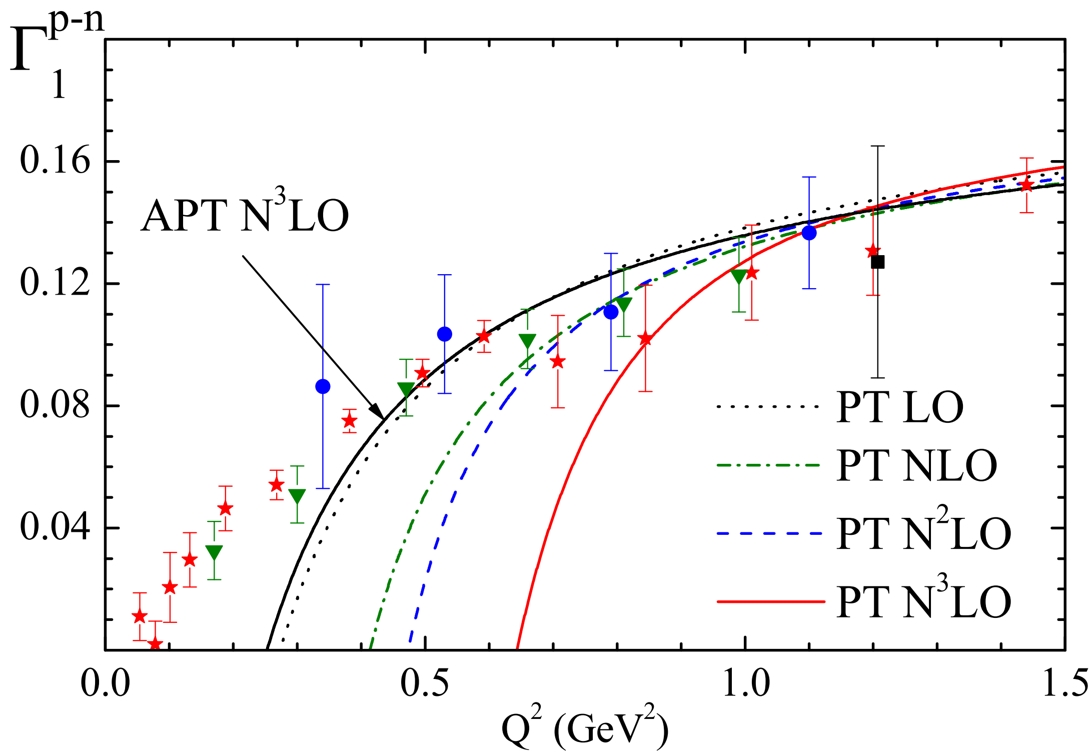
"Gluonic mass" calculated for few values of ξ.

ξ	Λ_1	$m_{gl}^{[1]}$	Λ_2	$m_{gl}^{[2]}$
8	244	690	324	915
10	249	787	315	995
12	253	876	305	1160

The given $m_{gl}^{[1,2]}$ values seem attractive as compared with the current lattice and other estimates.

The APT superiority over standard pQCD

The comparison of APT+1 twist with PT + 1 in JLab data description

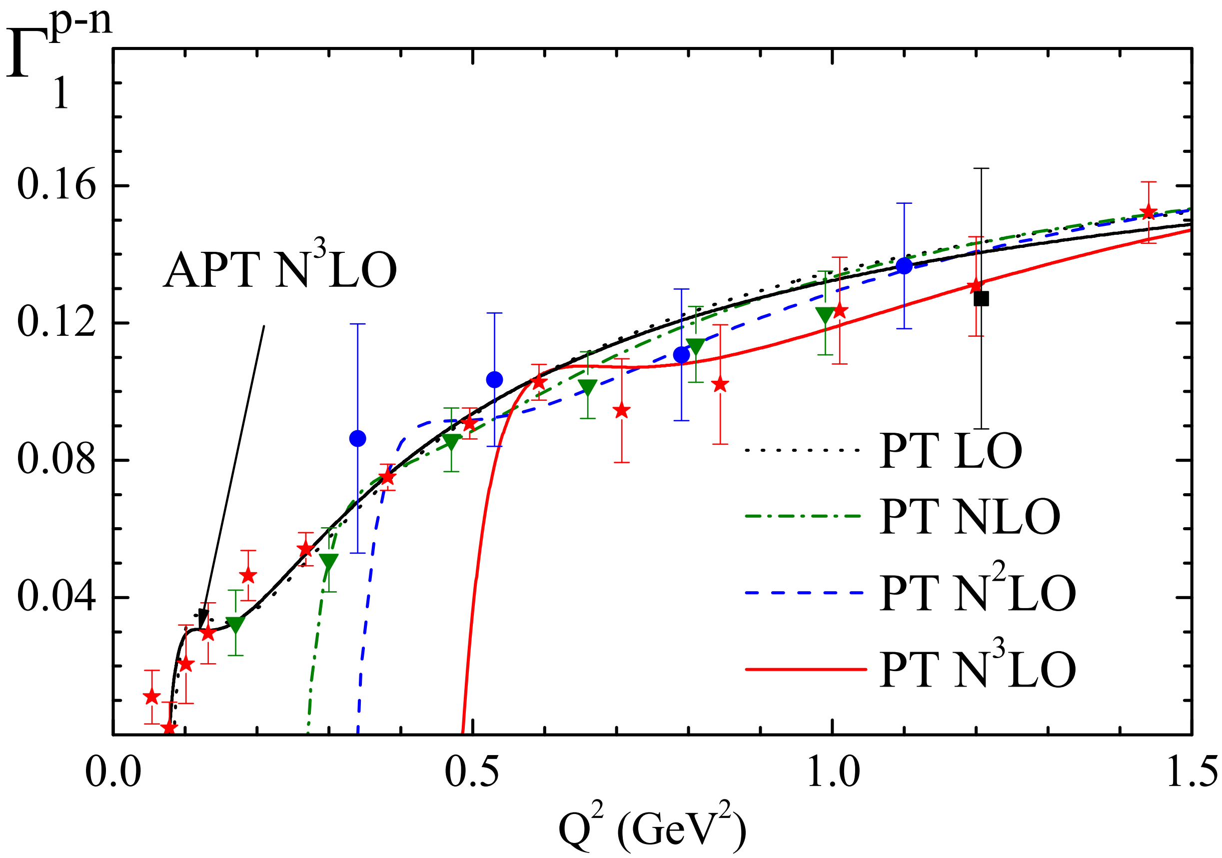


reveals that PT explodes, while **APT+1 HT works down to $Q \sim 500\,$MeV**.

MPT = "massive" pQCD

The APT superiority over pQCD, cont'd

The account for three HTs



does not care the PT catastrophe, while

APT+3HTs works better, down to $\mathbf{Q \sim 300}$ MeV .

MPT = "massive" pQCD

Comparing APT and MPT with the JLab data

Remind that for PT/APT we used 1- and 3-term HT sums

$$\Gamma_1(Q^2) = \frac{g_A}{6}\left[1 - \Delta^{PT/APT}(Q^2)\right] + \Gamma_{HT}^{(1)}\,; \quad \Gamma_{HT}^{(1)} = \frac{\mu_4}{Q^2}\,, \quad (16)$$

$$\Gamma_1(Q^2) = \frac{g_A}{6}\left[1 - \Delta^{PT/APT}(Q^2)\right] + \Gamma_{HT}^{(3)}\,; \quad \Gamma_{HT}^{(3)} \sum_{i=2}^{4} \frac{\mu_{2i}^{APT}}{Q^{2i-2}}\,. \quad (17)$$

For MPT case we insert <u>the HT generating function</u> $G(Q^2)$, regular at $Q^2 \geq 0$. [Khandramai+DVSh 12]

$$\Gamma_1^{MPT}(Q^2) = \frac{g_A}{6}\left[1 - \Delta^{MPT}\right] + G_{HT}(Q^2)\,; \quad \boxed{G_{HT}(Q^2) = \frac{\mu_4}{Q^2+m^2}}\,,$$

with $m \sim 700$ MeV. Now, Γ_1 is regular at $Q^2 \geq 0$.

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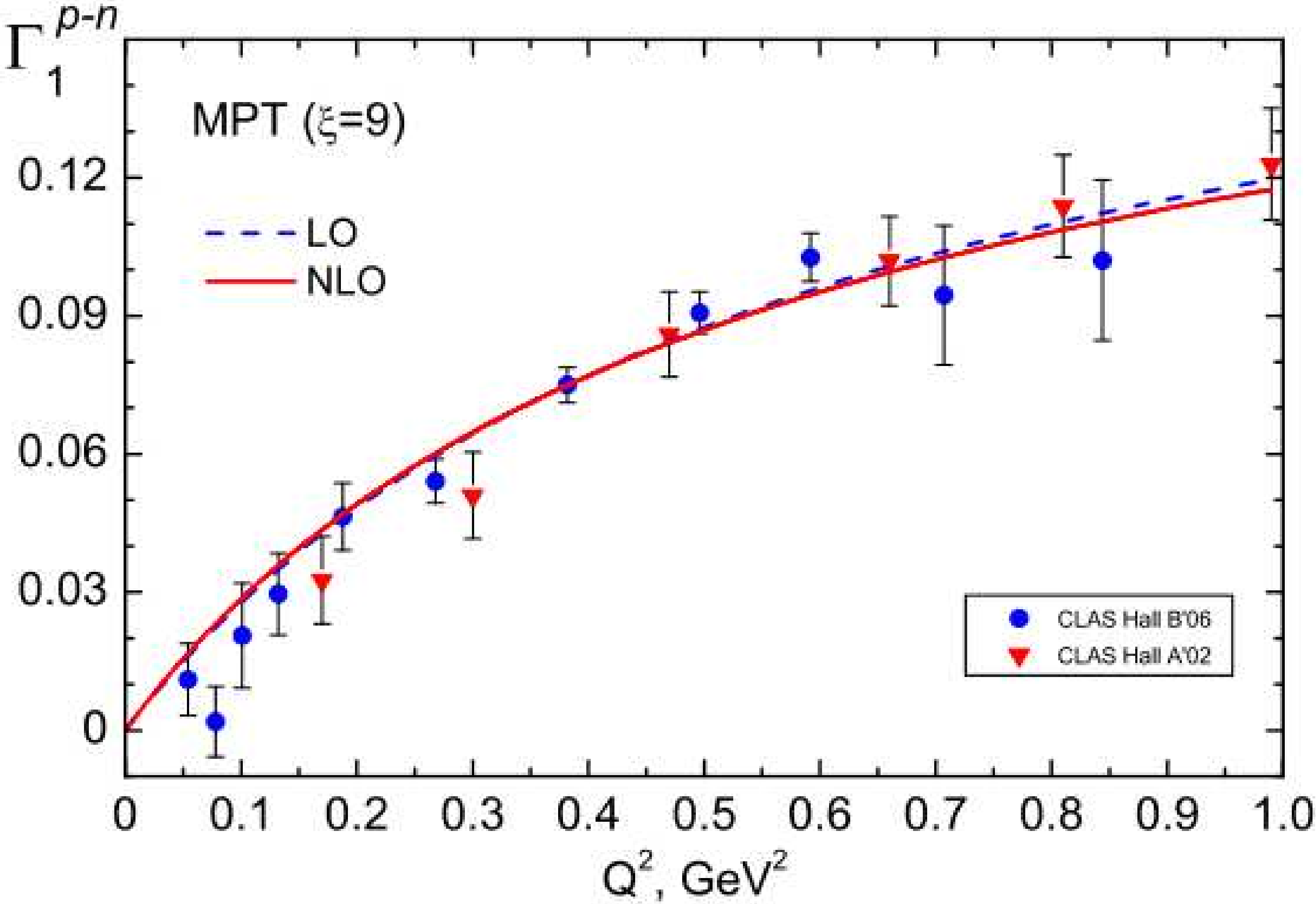
Comparing MPT+G_{HT} with the JLab data



MPT+G_{HT} works **down to the very IR limit !**

MPT = "massive" pQCD

Conclusion

- New pQCD danger : outbreak of asymptotic PT series at scale $Q, \sqrt{s} \lesssim 1\text{GeV}$, $\alpha_s \gtrsim 0.4$,
- The necessity of modification of usual running α_s, ("canonized" by PDG) in the LE region.
- Need of the MPT analysis of other LE processes

- The "gluonic" mass issue physical discussion...
- The HT terms issue and their generating function
- Correlation between HTs in various processes
- Interplay between "non-perturbative" and "perturbative" components

MPT = "massive" pQCD

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