# Higher Spins, Holography and String Theory

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- Higher Spin Field Theories have been one of fascinating and rapidly developing subjects over recent few years

- Higher spin fields constitute a crucial ingredient of AdS/CFT correspondence since they are presumably dual to multitudes of operators in the related CFT's.

- Higher Spin symmetries may also hold an important key to understanding of the true symmetries of gravity and unification models

- Despite significant progress in describing the dynamics of higher spin field theories, achieved over recent few decades, our understanding of the general structure of the higher spin interactions is still very far from complete

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One of the conceptual difficulties of constructing consistent gauge-invariant HS theories is related to the existence of the no-go theorems (such as Coleman-Mandula theorem)

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The no go theorems can, however, be circumvented in a number of cases, e.g. in the AdS space (where there is no well-defined S-matrix) also by relaxing some of constraints on locality etc.

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String theory appears to be a particularly efficient and natural framework to construct and analyze consistent gauge-invariant interactions of higher spins

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In my talk I review the basic concepts of string theory approach to analysis of higher spin interactions and the relation between vertex operator formalism in string theory and frame-like description of higher spin dynamics

# OUTLINE:

- Metric (Fronsdal) vs Frame-like Approaches to HS Field theories - brief review

- String Theory approach - Vertex Operator Construction for Massless Higher Spin connection Gauge Fields

- Higher Spin Interaction Vertices in Flat Space from String Theory Amplitudes

- Extension to AdS Space and Holography. String-theoretic Sigma-Model for HS dynamics in AdS.

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$AdS_4/CFT_3$ HS Holography and Liouville Field Theory.

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$AdS_5/CFT_4$ HS Holography and Fluid Dynamics. Higher Spins in $AdS_5$ as Vorticities in $D = 4$ Turbulence.

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Conclusion and Discussion

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In the simplest formulation, fields of spin $s$ are described by symmetric double traceless tensors of rank $s$ satisfying Pauli-Fierz on-shell conditions:

$$(\partial_m \partial^m + m^2) H_{n_1 \ldots n_s}(x) = 0$$
$$\partial^{n_1} H_{n_1 \ldots n_s}(x) = 0$$
$$\eta^{n_i n_j} \eta^{n_k n_l} H_{n_1 \ldots n_s}(x) = 0$$
$$(1 \leq i < j \leq s; 1 \leq k < l \leq s; i \neq j \neq k \neq l)$$

(from now on we will limit ourselves to the $m^2 = 0$ case) and the gauge symmetry

$$\delta H_{i_1 \ldots i_s}(x) = \partial_{(i_1} \Lambda_{i_2 \ldots i_s)(x)} \qquad (0.1)$$

where $\Lambda$ is symmetric and traceless. The gauge invariant free field action leading to (1), (2) has been first constructed by C. Fronsdal in 1978 and is given by:

$$S = \frac{1}{2}\int d^d x(\partial_m H_{n_1\dots n_s}\partial^m H^{n_1..n_s}$$
$$-\frac{1}{2}s(s-1)\partial_m H^n_{nn_3\dots n_s}\partial^m H_p^{pn_3..n_s}$$
$$+s(s-1)\partial_m H^n_{nn_3\dots n_s}\partial^p H_p^{mn_3..n_s}$$
$$-s\partial_m H^m_{n_2\dots n_s}\partial^n H_n^{n_2..n_s}$$
$$-\frac{1}{4}s(s-1)(s-2)\partial_m H^{mn}_{nn_3\dots n_s}\partial^p H_{pq}^{qn_3..n_s})$$

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This formalism, regarding $H$ as a metric-type object, is difficult to extend to the interacting case and/or to AdS geometry, although some limited progress was achieved in this direction. In particular, various examples of cubic interaction vertices in flat space were constructed in this formalism (e.g. Berends-Burgers-Van

Dam (1996);Boulanger-Bekaert-Cnockaert (2006); Sagnotti-Taronna (2010); Manvelyan,Mkrtchan,Ruhl 2009 etc.) However, to analyze the HS dynamics and HS symmetries in both flat and especially curved backgrounds such as AdS it is more natural to use the frame-like formalism developed by Vasiliev et.al. which turns out to be a powerful approach...

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Unlike the approach used by Fronsdal that considers higher spin tensor fields as metric-type objects, the frame-like formalism describes the higher spin dynamics in terms of higher spin connection gauge fields that generalize objects such as vielbeins and spin connections in gravity (in standard Cartan-Weyl formulation or Mac Dowell-Mansoury-Stelle-West (MMSW) in case of nonzero cosmological constant). The higher spin connections for a given spin $s$ are described by collection of two-row gauge fields (with the rows of lengths $s-1$ and $t$ accordingly)

$$\omega^{s-1|t} \equiv \omega_m^{a_1 \ldots a_{s-1}|b_1 .. b_t}(x)$$
$$0 \leq t \leq s-1$$
$$1 \leq a, b, m \leq d$$

traceless in the fiber indices, where $m$ is (generally) the curved $d$-dimensional space index while $a, b$ label the tangent space with $\omega$ satisfying

$$\omega_m^{(a_1 \ldots a_{s-1}|b_1) .. b_t} = 0$$

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$$\omega_m^{(a_1 \ldots a_{s-1}|b_1)..b_t} = 0$$

The gauge transformations for $\omega$ are given by

$$\omega_m^{a_1 \ldots a_{s-1}|b_1..b_t} \rightarrow \omega_m^{a_1 \ldots a_{s-1}|b_1..b_t} + D_m \rho^{a_1 \ldots a_{s-1}|b_1..b_t}$$

while the diffeomorphism symmetries are

$$\omega_m^{a_1...a_{s-1}|b_1..b_t}(x) \to \omega_m^{a_1...a_{s-1}|b_1..b_t}(x)$$
$$+\partial_m\epsilon^n(x)\omega_n^{a_1...a_{s-1}|b_1..b_t}(x)$$
$$+\epsilon^n(x)\partial_n\omega_m^{a_1...a_{s-1}|b_1..b_t}(x)$$

The $\omega^{s-1|t}$ gauge fields with $t \geq 0$ are auxiliary fields related to the dynamical field $\omega^{s-1|0}$ by generalized zero torsion constraints:

$$\omega_m^{a_1...a_{s-1}|b_1...b_t} \sim \partial^{b_1}...\partial^{b_t}\omega_m^{a_1...a_{s-1}}$$

skipping pure gauge terms (for convenience of the notations, we set the cosmological constant to 1, anywhere the $AdS$ backgrounds are concerned)

It is also convenient to introduce the $d+1$-dimensional index $A = (a, \hat{d})$ (where $\hat{d}$ labels the extra dimension) and to combine $\omega^{s|t}$ into a single two-row field $\omega^{A_1...A_{s-1}|B_1...B_{s-1}}(x)$

identifying

$$\omega^{s-1|t} = \omega^{a_1\ldots a_{s-1}|b_1\ldots b_t\hat{d}\ldots\hat{d}}$$

$$\omega^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}} V_{A_{t+1}}\ldots V_{A_{s-1}}$$
$$= \omega^{A_1\ldots A_{s-1}|B_1\ldots B_t}$$

where $V_A$ is the compensator field satisfying $V_A V^A = 1$. The Fronsdal field $H^{a_1\ldots,a_s}$ is then obtained by symmetrizing $\omega^{(a_1\ldots,a_s)} = e^{m(a_s}\omega_m^{a_1\ldots a_{s-1})}$.

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The generalized HS curvature is defined according to

$$R^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}} = d\omega^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}}$$
$$+(\omega \wedge \star\omega)^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}}$$

where $\star$ is the associative product in higher spin symmetry algebra. The explicit structure of this product depends on the basis chosen and in general is quite complicated The HS dynamics is then described by EOM

$$R^{A_1 \ldots A_{s-1}|B_1 \ldots B_{s-1}} V_{B_1} \ldots V_{B_{s-1}} = 0$$

## HS VERTEX OPERATORS: PRELIMINARIES

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We now turn to the questions of constructing vertex operators for the higher spin connection gauge fields in open RNS superstring theory. The strategy is that

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BRST invariance conditions on these operators leads to Pauli-Fierz on-shell constraints

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BRST nontriviality: Gauge symmetry transformations on $\omega^{s-1|t}$ higher spin connection gauge fields leads to shifting the vertex operators by BRST-exact terms. The correlation functions of the vertex operators for the frame-like fields are therefore gauge-invariant by construction.

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The worldsheet N-point correlators of the operators determine polynomial degree N interactions of the HS fields in the frame-like formalism. In AdS backgrounds, these interactions correspond to $N$-point correlations in dual CFT's.

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In string theory the physical states are described by physical BRST non-trivial and BRST-invariant vertex operators. In the zero momentum limit these operators are closely related to generators of global space-time symmetries. For example, the photon (spin 1) vertex operator

$$V_{ph} = A_m(p) \oint dz(\partial X^m + i(p\psi)\psi^m)e^{ipX}(z)$$

reduces to translation generator of Poincare algebra. It is convenient to unify the Poincare generators ($T_a$,$T_{ab}$) into 1-form:

$$\Omega = (e^a_m T_a + \omega^{ab}_m T_{ab})dx^m$$

where $e^a_m$ and $\omega^{ab}_m$ are $s = 2$ vielbein and spin connection, i.e. the $\omega^{1|0}$ and $\omega^{1|1}$ components of $\omega^{A|B}_m$. Given the Poincare commutation relations, $R = d\Omega + \Omega \wedge \Omega$ reproduces the standard Lorenz curvature tensor for spin 2 describing gravitational fluctuations around the flat vacuum (for Poincare replaced by AdS isometry algebra one obtains Riemann's tensor shifted by appropriate cosmological terms)

The higher spin generalization of $\Omega$ 1-form is

$$\Omega = dx^m(e_m^a T_a + \omega_m^{ab} T_{ab}$$
$$+ \sum_s \sum_{t=0}^{s-1} \omega_m^{a_1 \ldots a_{s-1}|b_1 \ldots b_t} T_{a_1 \ldots a_{s-1}|b_1 \ldots b_t})$$

where $\omega_m^{a_1 \ldots a_{s-1}|b_1 \ldots b_t}$ are higher spin connections and $T_{a_1 \ldots a_{s-1}|b_1 \ldots b_t}$ are the HS algebra generators. For this reason, we expect the vertex operators for the frame-like fields to be related to generators of HS space-time symmetry algebra, i.e. the HS algebra is realized as an operator algebra of the vertices.

# HS VERTEX OPERATORS: CONSTRUCTION

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The RNS superstring action is given by

$$S_{RNS} = S_{matter} + S_{b-c} + S_{\beta-\gamma}$$
$$S_{matter} = -\frac{1}{2\pi}\int d^2z\{\partial X_m \bar{\partial} X^m(z,\bar{z})$$
$$+\bar{\partial}\psi_m\psi^m + \partial\bar{\psi}_m\bar{\psi}^m\}$$
$$Sb - c = \int d^2z\{b\bar{\partial}c + \bar{b}\partial\bar{c}\}$$
$$S_{\beta-\gamma} = \int d^2z\{\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma}\}$$

and the bosonization relations for the fermionic and bosonic ghosts are

$$b(z) = e^{-\sigma}; c = e^{\sigma}(z)$$
$$\gamma(z) = e^{\phi-\chi}(z) \equiv e^{\phi}\eta(z)$$
$$\beta(z) = e^{\chi-\phi}\partial\chi \equiv e^{-\phi}\partial\xi(z)$$

and similarly for $\bar{b}, \bar{c}, \bar{\beta}, \bar{\gamma}$. The nature of the vertex operators for the frame-like fields that we shall propose below, is very different from the standard RNS vertices like that of a photon. As it is well known, the photon operator $V_0 \sim A_m(p) \oint dz(\partial X^m + (p\psi)\psi^m)e^{ipX}$ (where $p^2 = (pA(p)) = 0$) can be also represented as at any integer superconformal ghost picture $n$ with the representations at different pictures related according to

$$V_n =: \Gamma V_{n-1} :\equiv \{Q, \xi V_{n-1}\}$$
$$V_{n-1} =: \Gamma^{-1} V_n :$$
$$: \Gamma^{-1}\Gamma := 1$$

where

$$\Gamma =: e^{\phi} G :\equiv \{Q, \xi\}$$
$$\Gamma^{-1} = -4c\partial\xi e^{-2\phi}$$

are the direct and inverse picture changing operators

$$G = -\frac{1}{2}b\gamma + \frac{3}{2}\beta\partial c + \partial\beta c$$

is the full matter+ghost worldsheet supercurrent and

$$Q = \oint dz\{cT - bc\partial c$$
$$-\frac{1}{2}\gamma\psi_m\partial X^m - \frac{1}{4}\gamma^2 b\}$$

For example, for a photon

$$V_{-2} = A_m(p)\oint dze^{-2\phi}\partial X^m e^{ipX}$$
$$V_{-1} = A_m(p)\oint dze^{-\phi}\psi^m e^{ipX}$$
$$V_0 = A_m(p)\oint dz(\partial X^m + (p\psi)\psi^m)e^{ipX}$$

The vertex operators for the higher spin connection gauge fields are different, as they violate the picture equivalence and their coupling to $\beta - \gamma$ system is essential and can be classified in terms of superconformal ghost cohomologies $H_n$.

Definition and Properties:

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Positive ghost cohomologies $H_n (n \geq 1)$ consist of physical (BRST invariant and nontrivial) vertex operators that exist at pictures n and above (related by standard transformations with $\Gamma$ and $\Gamma^{-1}$) and are annihilated by $\Gamma^{-1}$ at the minimal positive ghost picture $n$.

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Negative ghost cohomologies $H_n (n \leq -3)$ consist of physical (BRST invariant and nontrivial) vertex operators that exist at pictures n and below (related by standard transformations with $\Gamma$ and $\Gamma^{-1}$) and are annihilated by direct picture changing $\Gamma$ at the minimal positive ghost picture $n$

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There is an isomorphism between positive and negative ghost cohomologies: $H_n \sim H_{-n-2}; n \geq 1$ as any element $V^{(-n-2)}$ of $H_{-n-2}$ is related to to the corresponding element element $V^{(n)}$ of $H_n$ by transformation: $V^{(n)} \sim: Z\Gamma^{2n+2} : V^{(-n-2)}$ where $Z =: b\delta(T) :$ is the p.c. operator for the $b-c$ ghost fields (SUSY analogue of $\Gamma =: \delta(\beta)G :$ which is the p.c. operator for the $\beta - \gamma$ system). Therefore, each element of $H_n$ has the negative picture mirror in $H_{-n-2}$ with the identical on-shall and gauge-invariance conditions for the space-time fields.

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The vertex operators for higher spin frame-like fields are the elements of $H_n \sim H_{-n-2}$ with $n+2$ roughly corresponding to the spin value.

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OPE fusion rules for ghost cohomologies of different ranks are similar to the HS algebraic structure for generators with different spin values (including truncation properties)

The spin 3 operator for $\omega^{2|0}$ dynamic field is given by

$$V^{(-3)} = H_{abm}(p)ce^{-3\phi}\partial X^a \partial X^b \psi^m e^{ipX}$$

at unintegrated $H_{-3}$-representation and

$$V^{(+1)} = K \circ H_{abm}(p) \oint dz e^{\phi}\partial X^a \partial X^b \psi^m e^{ipX}$$

at integrated $H_1$-representation The homotopy transformation $K \circ T$ of an integrated operator $T = \oint dz V(z)$ (with $V(z)$ being a primary field of dimension 1) is defined according to

$$K \circ T =$$
$$T + \frac{(-1)^N}{N!} \oint \frac{dz}{2i\pi} (z-w)^N : K\partial^N W : (z)$$
$$+ \frac{1}{N!} \oint \frac{dz}{2i\pi} \partial_z^{N+1} [(z-w)^N K(z)] K\{Q_{brst}, U\}$$

where

$$K = -4ce^{2\chi - 2\phi}$$

is homotopy operator satisfying

$$\{Q, K\} = 1$$

$U$ and $W$ are the operators appearing in the commutator

$$[Q, V(z)] = \partial U(z) + W(z)$$

so

$$[Q, \oint dz V(z)] = W \qquad (0.2)$$

It is the easiest to choose the negative cohomology representation for the BRST analysis.

The BRST-invariance constraint on the spin 3 operator leads to Pauli-Fierz type conditions

$$p^2 H_{abm} = p^a H_{abm} = \eta^{ab} H_{abm} = 0$$

However, in general

$$\eta^{am} H_{abm} \neq 0$$

as the tracelessness in $a$ and $m$ or $b$ and $m$ indices isn't required for $V^{(-3)}$ to be primary field. In what follows below we shall interpret $H_{abm}$ with the dynamical spin 3 connection form $\omega^{2|0}$, identifying $m$ with the manifold index and $a, b$ with the fiber indices. So the tracelessness condition is generally imposed by BRST invariance constraint on any pair of fiber indices only (but not on a pair of manifold and fiber indices). The same is actually true also for the vertex operators for frame-like gauge fields of spins higher than 3. Altogether, this corresponds precisely to the double tracelessness constraints for corresponding metric-like Fronsdal's fields for higher spins (although the zero double trace condition does not of course appear in the case of $s = 3$) As it is clear from the manifest expressions, the tensor $H_{abm}$ is by definition symmetric in indices $a$ and $b$ and therefore can be represented as a sum of two Young diagrams. However, only the fully symmetric diagram is the physical state, since the second one (with two rows) can be represented as the BRST commutator in the small Hilbert space:

$$V^{(-3)} \sim \{Q, W\}$$

$$W = H_{abm}(p) c\partial\xi e^{-4\phi+ipX}\partial X^a(\psi^{[m}\partial^2\psi^{b]} \\ -2\psi^{[m}\partial\psi^{b]}\partial\phi \\ +\psi^m\psi^b(\frac{5}{13}\partial^2\phi + \frac{9}{13}(\partial\phi)^2))$$

$$+a \leftrightarrow b$$

Of course everything described above also applies to the vertex operator at positive cohomology, with appropriate $Z, \Gamma$ transformations. This altogether already sends a strong hint to relate the operators for $H_{abm}$ to vertex operators for the dynamical frame-like field $\omega^{2|0}$ describing spin 3. However, to make the relation between string theory and frame-like formalism precise, we still need the vertex operators for the remaining extra fields $\omega^{2|1}$ and $\omega^{2|2}$. The expressions that we propose are given by

$$V^{2|1}(p) = 2\omega_m^{ab|c}(p)ce^{-4\phi} \times$$
$$(-2\partial\psi^m\psi_c\partial X_{(a}\partial^2 X_{b)}$$
$$-2\partial\psi^m\partial\psi_c\partial X_a\partial X_b$$
$$+\psi^m\partial^2\psi_c\partial X_a\partial X_b)e^{ipX}$$

for $\omega^{2|1}$ and

$$V^{2|2}(p) = -3\omega_m^{ab|cd}(p)ce^{-5\phi} \times$$
$$(\psi^m\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b$$
$$-2\psi^m\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b$$
$$+\frac{5}{8}\psi^m\partial\psi_c\partial^2\psi_d\partial X_a\partial^3 X_b$$
$$+\frac{57}{16}\psi^m\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b)e^{ipX}$$

for $\omega^{2|2}$. We start with analyzing the operator for $\omega^{2|1}$ Straightforward application of $\Gamma$ to this operator gives

$$: \Gamma V^{2|1} : (p) = V^{(-3)}(p) H_m^{ab}(p)$$
$$= i p_c \omega_m^{ab|c}(p) \qquad (0.3)$$

i.e. the picture-changing of $V^{2|1}$ gives the vertex operator for $\omega^{2|0}$ with the 3-tensor given by the divergence of $\omega^{2|1}$, i.e. for $p_c\omega_m^{ab|c}(p) \neq 0$ $V^{2|1}$ is the element of $H_{-3}$. If,however, the divergence vanishes, the cohomology rank changes and $V^{2|1}$ shifts to $H_{-4}$. This is precisely the case we are interested in. Namely, consider the $H_{-4}$ cohomology condition

$$p_c\omega_m^{ab|c}(p) = 0$$

The general solution of this constraint is

$$\omega_m^{ab|c} = 2p^c\omega_m^{ab} - p^a\omega_m^{bc}$$
$$-p^b\omega_m^{ac} + p_d\omega_m^{acd;b}$$

where $\omega_m^{ab}$ is traceless and divergence free in $a$ and $b$ and satisfies the same on-shell constraints as $H_m^{ab}$, while $\omega_m^{acd;b}$ is some three-row field, antisymmetric in $a, c, d$ and symmetric in $a$ and $b$. It is, however, straightforward to check that the operator $V^{2|1}$ with the polarization given by $\omega^{ab|c} = p_d \omega_m^{acd;b}$ can be cast as the BRST commutator:

$$
\begin{aligned}
& p_d \omega_m^{acd;b}(p) V_{ac|b}^m(p) \\
&= \{Q, \omega_m^{acd;b}(p) \oint dz e^{\chi - 5\phi + ipX} \partial\chi \\
&\times(-2\partial\psi^m \psi_c \partial X_a \partial^2 X_b \\
&-2\partial\psi^m \partial\psi_c \partial X_a \partial X_b \\
&+\psi^m \partial^2\psi_c \partial X_a \partial X_b) \\
&\times(\partial^2\psi_d - \frac{4}{3}\partial\psi_d \partial\phi \\
&+\frac{1}{141}\psi_d(41(\partial\phi)^2 - 29\partial^2\phi))\}
\end{aligned}
$$

therefore, modulo pure gauge terms the cohomology condition on $V_{2|1}$ is the zero torsion condition relating the extra field $\omega^{2|1}$ to the dynamical $\omega^{2|0}$ connection. Similarly, the $H_{-5} \sim H_3$ cohomology condition on $V_{2|2}$ and $\omega^{2|2}$ leads to generalized zero torsion constraints relating $\omega^{2|2}$ to $\omega^{2|1}$ and $\omega^{2|0}$. These are the second generalized zero torsion condition given by

$$\omega_m^{ab|cd} = 2p^d \omega^{ab|c} - p^a \omega^{bd|c}$$
$$-p^b \omega^{ad|c} + 2p^c \omega^{ab|d} - p^a \omega^{bc|d} - p^b \omega^{ac|d}$$

relating $\omega^{2|2}$ to $\omega^{2|1}$ modulo BRST-exact terms $\sim \{Q, W^{2|2}(p)\}$ where

$$W^{2|2}(p) = \omega^{ab;cdf}(p) \oint dz e^{ipX} [$$
$$\times (\psi^m \partial^2 \psi_c \partial^3 \psi_d \partial X^a \partial X_b$$

$$-2\psi^m\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b$$
$$+\frac{5}{8}\psi^{(m}\partial\psi_c\partial^2\psi_{d)}\partial X_a\partial^3 X_b$$
$$+\frac{57}{16}\psi^m\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b)$$
$$\times(-\frac{5}{2}L_f\partial^2\xi+\partial L_f\partial\xi)$$

where, as previously, $\xi = e^\chi$ and

$$L_f = e^{-6\phi}(\partial^2\psi_f - \partial\psi_f\partial\phi$$
$$+\frac{3}{25}\psi_f((\partial\phi)^2 - 4\partial^2\phi))$$

.

Similarly,

The gauge transformation for the $\omega^{2|1}$ field:

$$\omega_m^{ab|c}(p) \to \omega_m^{ab|c}(p) + p_m\Lambda^{ab|c}(p)$$

leads to shifting the $V^{2|1}$ vertex operator (21) by BRST-exact terms:

$$V^{2|1}(p) \to V^{2|1}(p) + \{Q, W_1^{2|1}(p)\}$$

where, up to overall numerical factor,

$$W_1^{2|1}(p) \sim \Lambda^{ab|c}(p) \oint dzce^{-5\phi+ipX}$$
$$\times((p\partial\psi)(\psi_c\partial^2 X_b - 2\partial\psi_c\partial X_b)$$
$$+(p\psi)\partial^2\psi_c\partial X_b)$$
$$\times(\frac{2}{5}\partial L_a\partial\xi - L_a\partial^2\xi)$$

where

$$L_a = \partial^2\psi_a - 2\partial\psi_a\partial\phi$$
$$+\frac{1}{13}\psi_a(5\partial^2\phi + 9(\partial\phi)^2)$$

and $\Lambda$ has the same symmetry in the fiber indices as $\omega^{2|1}$. This operator is BRST-exact if

$\omega$ is transverse in the $a, b$ fiber indices (which, in turn, is the invariance condition). Next, if $\omega_m^{ab|c}(p)$ is antisymmetric in $m$ and $a$ (so that the corresponding $\omega^{2|0}$ is the two-row field), $V^{2|1}$ is again the BRST commutator in the small Hilbert space:

$$V^{2|1}(p) = \{Q, W_2^{2|1}(p)\}$$

where

$$\begin{aligned} W_2^{2|1}(p) \sim \omega_m^{ab|c}(p) \oint dzce^{-5\phi+ipX} \\ \times(\psi_c\partial^2 X_b - \partial\psi_c\partial X_b) \\ \times(\frac{2}{5}\partial\psi^{[m}\partial L_{a]}\partial\xi - \partial\psi^{[m}L_{a]}\partial^2\xi) \\ +\partial^2\psi_c\partial X_b(\frac{2}{5}\psi^{[m}\partial L_{a]}\partial\xi \\ -\psi^{[m}L_{a]}\partial^2\xi) \end{aligned}$$

Next, we analyze $\omega^{2|2}$ and its vertex operator. The gauge transformation for the $\omega^{2|2}$ field:

$$\omega_m^{ab|cd}(p) \to \omega_m^{ab|cd}(p) + p_m \Lambda^{ab|cd}(p)$$

leads to shifting the $V^{2|2}$ vertex operator (21) by BRST-exact terms:

$$V^{2|2}(p) \to V^{2|2}(p) + \{Q, W_1^{2|2}(p)\}$$

with

$$W_2^{2|2}(p) \sim \Lambda^{ab|cd}(p) \oint dzce^{-6\phi+ipX}$$
$$\times\{(\frac{1}{4}(p_n\partial N^n)\partial\xi - (p_n N^n)\partial^2\xi)$$
$$\times(\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b$$
$$-2\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b$$
$$+\frac{5}{8}\partial\psi_c\partial^2\psi_d\partial X_a\partial^3 X_b$$
$$+\frac{57}{16}\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b)\}$$

where

$$N_n = \partial^3 X_n - \frac{3}{2}\partial^2 X_n$$
$$-\frac{1}{3}\partial X_n((\partial\phi)^2 - \frac{17}{6}\partial^2\phi)$$

As before, this operator is BRST-exact if $\omega$ is transverse in the $a, b$ fiber indices. Finally, if $\omega_m^{ab|cd}(p)$ is antisymmetric in $m$ and $a$ or $b$ (so that the corresponding $\omega^{2|0}$ is the two-row field), $V^{2|2}$ is again the BRST commutator in the small Hilbert space:

$$V^{2|2}(p) = \{Q, W_2^{2|2}(p)\}$$

with

$$W_2^{2|2}(p) \sim \omega_m^{ab|cd}(p) \oint dz c e^{-6\phi+ipX}$$
$$\{(\frac{1}{4}N^m \partial\xi - (N^m)\partial^2\xi)$$
$$\times(\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b$$

$$-2\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b$$
$$+\frac{5}{8}\partial\psi_c\partial^2\psi_d\partial X_a\partial^3 X_b$$
$$+\frac{57}{16}\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b)$$
$$-(a\leftrightarrow m)\}$$

It is now straightworward to compute 3-point cubic vertex for $s = 3$ Note that, string theoretic computation in the Fronsdal's formalism would be impossible to apply to $s = 3$ cubic vertex in a straightforward way sinse

$$H_1 \otimes H_2 \sim H_0 \oplus H_2$$

while Fronsdal-type correlator for $s = 3$ cubic vertex would be of the type $< H_1 H_1 H_1 >$

**Therefore the string-theoretic formalism must be combined with frame-like**

**description in this computation** Since

$$\omega^{2|1}V_{2|1} \subset H_2$$

$$H_1 \otimes H_1 \sim H_2 + ...$$

the relevant correlator is given by

$$A(p,k,q) =<: \Gamma^2 V_{2|1} : (p)V_{2|0}(k)V_{2|0}(q) >$$

where the double picture changing transformation of $V_{2|1}$ operator for $\omega^{2|1}$ frame-like field is needed to ensure the correct ghost number balance ( any correlator must carry total ghost $\phi$- number $-2$, ghost $\chi$- number $+1$, and ghost $\sigma$- number $+2$, in order to ensure the cancellation of $b, c, \beta, \gamma$ background charges) Note that the ghost balance conditions crucially control the derivative structure of $HS$ interactions and also the $HS$ algebraic structure both in flat and AdS backgrounds. The final answer for the 3-point spin 3 interaction vertex is

$$A(p,k,q) = \frac{691072283467i}{720}$$
$$\times\omega_n^{s_1s_2}(p)\omega_p^{t_1t_2}(k)\omega_m^{ab|cd}(q)$$
$$\times\{\eta^{nm}\eta_{pd}(\frac{1}{36}\eta^{s_1a}\eta^{s_2b}\eta^{t_1c}q^{t_2}$$
$$+\frac{4}{3}\eta^{t_1a}\eta^{s_1b}\eta^{t_2c}k^{s_2}$$
$$+\frac{1}{12}\eta^{s_1t_1}\eta^{s_2a}\eta^{t_2b}k^c$$
$$-\eta^{s_1t_1}\eta^{s_2a}\eta^{t_2c}p^b) + Symm(m,a,b)\}$$

This , up to total derivative terms and overall normalization factor, reproduces the well-known BBD (Berends,Burgers, Van Dam) 3-derivative interaction vertex for spin 3 fields Using this formalism, it is also straightforward to calculate new (so far, unknown) gauge invariant quartic interactions of spin 5 - spin 1, spin 3-spin 1 , along with gravitational interactions of spin 3 and to extend these calculations for AdS case (D.P.,Phys.Rev. D82 (2010) 066005, Phys.Rev. D83 (2011) 046005, Phys.Rev. D84 (2011) 126004; Seunjing Lee and D.P., 1203.0909 , D.P. and Soo-Jong Rey, in progress)

## Discussion and Outline

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The entire subject is a rapidly developing field with many exciting prospects, opportunities an fascinating problems and mysteries to resolve.

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A very subjective and incomplete of questions to address in the near future includes:

- Usin string-theoretic approach for constructing consistent gauge-invariant interactions of fields with spin greater than 3 as well as higher order (quartic, quintic etc) and to extend these results to AdS backgrounds

- Developing an OPE string-theoretic approach to AdS higher spin algebras in order to understand sequence of holographies existing in the Universe (ADT conjecture, AdS/CFT may be just particular examples of certain far more general principle relating field and string theories in various dimensions

- Higher derivative interactions in HS field theories vs. higher derivative expansion in hydrodynmics; Strings/HS versus Gravity/Fluid dynamics and AdS/CMT?

- Higher Spins as a secret key to non-SUSY holography; a brave new world but nothing new under the Sun!