Tubelike Wormholes and Charge Confinement

Talk at the 7th Mathematical Physics Meeting, Belgrade, Sept 09-19, 2012

Eduardo Guendelman[1], Alexander Kaganovich[1], Emil Nissimov[2], Svetlana Pacheva[2]

[1] Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel

[2] Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

Background material and further development of:

- E. Guendelman, A. Kaganovich, E.N. and S. Pacheva,
  (1) *"Asymptotically de Sitter and anti-de Sitter Black Holes with Confining Electric Potential"*, *Phys. Lett.* **B704** (2011) 230-233, erratum *Phys. Lett.* **B705** (2011) 545 ;
  (2) *"Hiding Charge in a Wormhole". The Open Nuclear and Particle Physics Journal* **4** (2011) 27-34 (*arxiv:1108.3735*[hep-th]);
  (3) *"Hiding and Confining Charges via 'Tubelike' Wormholes". Int.J. Mod. Phys.* **A26** (2011) 5211-5239;
  (4) *"Dynamical Couplings, Dynamical Vacuum Energy and Confinement/Deconfinement from R^2-Gravity".* *arxiv:1207.6775*[hep-th].

Introduction - Overview of Talk

We consider gravity (including $f(R)$-gravity) coupled to **non-standard** nonlinear gauge field system containing $-\frac{f_0}{2}\sqrt{-F^2}$. The latter is known to produce in flat space-time a **QCD-like confinement**.

Several interesting features:

- New mechanism for **dynamical** generation of cosmological constant due to nonlinear gauge field dynamics: $\Lambda_{\rm eff} = \Lambda_0 + 2\pi f_0^2$ (Λ_0 – bare CC, may be absent at all) ;
- Non-standard black hole solutions of Reissner-Nordström-(anti-)de-Sitter type containing a **constant radial vacuum electric field** (in addition to the Coulomb one), in particular, in electrically neutral black holes of Schwarzschild-(anti-)de-Sitter type;

Introduction - Overview of Talk

- In case of vanishing effective cosmological constant $\Lambda_{\rm eff}$ (*i.e.*, $\Lambda_0 < 0\ ,\ |\Lambda_0| = 2\pi f_0^2$) the resulting Reissner-Nordström-type black hole, apart from carrying an additional constant vacuum electric field, turns out to be **non-asymptotically flat** – a feature resembling the gravitational effect of a **hedgehog**;
- Appearance of **confining-type effective potential** in charged test particle dynamics in the above black hole backgrounds;
- New "tubelike" solutions of Levi-Civita-Bertotti-Robinson type, *i.e.*, with space-time geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is a two-dimensional anti-de Sitter, Rindler or de Sitter space depending on the relative strength of the electric field w.r.t. the coupling f_0 of the square-root gauge field term.

Introduction - Overview of Talk

When in addition one or more **lightlike branes** are self -consistently coupled to the above gravity/nonlinear-gauge-field system (as matter and charge sources) they produce ("thin-shell") wormhole solutions dislaying two novel physically interesting effects:

- **"Charge-hiding" effect** - a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer – a phenomenon opposite to the famous Misner-Wheeler "charge without charge" effect;
- **Charge-confining "tubelike" wormhole** with two "throats" occupied by two oppositely charged lightlike branes – the whole electric flux is confined within the finite-extent "middle universe" of generalized Levi-Civita-Bertotti-Robinson type – no flux is escaping into the outer non-compact "universes".

Additional interesting features appear when we couple the "square-root" confining nonlinear gauge field system to $f(R)$-gravity with $f(R) = R + \alpha R^2$ and a dilaton. Reformulating the model in the physical "Einstein" frame we find:

- **"Confinement-deconfinement" transition** due to appearance of "flat" region in the effective dilaton potential;
- The effective gauge couplings as well as the induced cosmological constant become **dynamical** depending on the dilaton v.e.v. In particular, a conventional Maxwell kinetic term for the gauge field is **dynamically generated** even if absent in the original theory;

Introduction - Overview of Talk

- **Regular black hole** solution (*no singularity* at $r = 0$) **with confining vacuum electric field**: the bulk space-time consist of two regions – an interior de Sitter and an exterior Reissner-Nordström-type (with “hedgehog asymptotics”) glued together along their common horizon occupied by a charged lightlike brane. The latter also dynamically determines the non-zero cosmological constant in the interior de-Sitter region.

Why $\sqrt{-F^2}$?

‘t Hooft has shown that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as an “infrared counterterm”).
The simplest way to realize these ideas in flat space-time:

$$S = \int d^4x L(F^2) \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \; , \tag{1}$$
$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; ,$$

The square root of the Maxwell term naturally arises as a result of **spontaneous breakdown of scale symmetry** of the original scale-invariant Maxwell action with f_0 appearing as an integration constant responsible for the latter spontaneous breakdown

Introduction - Motivation for $\sqrt{-F^2}$

For static field configurations the model (1) yields an electric displacement field $\vec{D} = \vec{E} - \frac{f_0}{\sqrt{2}} \frac{\vec{E}}{|\vec{E}|}$ and the corresponding energy density turns out to be $\frac{1}{2}\vec{E}^2 = \frac{1}{2}|\vec{D}|^2 + \frac{f_0}{\sqrt{2}}|\vec{D}| + \frac{1}{4}f_0^2$, so that it indeed contains a term linear w.r.t. $|\vec{D}|$. The model (1) produces, when coupled to quantized fermions, a confining effective potential $V(r) = -\frac{\beta}{r} + \gamma r$ (Coulomb plus linear one with $\gamma \sim f_0$) which is of the form of the well-known “Cornell” potential in the phenomenological description of quarkonium systems in QCD.

Gravity Coupled to Confining Nonlinear Gauge Field

The action (R-scalar curvature; Λ_0 - bare CC, might be absent):

$$S = \int d^4x \sqrt{-G} \Big[\frac{R - 2\Lambda_0}{16\pi} + L(F^2) \Big] \quad , \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f_0}{2} \sqrt{-F^2} \, ,$$

$$F^2 \equiv F_{\kappa\lambda} F_{\mu\nu} G^{\kappa\mu} G^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \, .$$

The corresponding equations of motion read – Einstein eqs.:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \Lambda_0 G_{\mu\nu} = 8\pi T^{(F)}_{\mu\nu} \, , \tag{3}$$

$$T^{(F)}_{\mu\nu} = \left(1 - \frac{f_0}{\sqrt{-F^2}}\right) F_{\mu\kappa} F_{\nu\lambda} G^{\kappa\lambda} - \frac{1}{4} \left(F^2 + 2 f_0 \sqrt{-F^2}\right) G_{\mu\nu} \, , \tag{4}$$

and nonlinear gauge field eqs.:

$$\partial_\nu \left(\sqrt{-G} \left(1 - \frac{f_0}{\sqrt{-F^2}}\right) F_{\kappa\lambda} G^{\mu\kappa} G^{\nu\lambda} \right) = 0 \, . \tag{5}$$

Static Spherically Symmetric Solutions

Non-standard Reissner-Nordström-(anti-)de-Sitter-type black holes depending on the sign of the dynamically generated CC $\Lambda_{\rm eff}$:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \; , \quad (6)$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\rm eff}}{3}r^2 \quad , \quad \Lambda_{\rm eff} = 2\pi f_0^2 + \Lambda_0 \; , \quad (7)$$

with static spherically symmetric electric field containing apart from the Coulomb term an additional *constant* “vacuum” piece:

$$F_{0r} = \frac{\varepsilon_F f_0}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\, r^2} \quad , \quad \varepsilon_F \equiv \mathrm{sign}(F_{0r}) = \mathrm{sign}(Q) \; , \quad (8)$$

corresp. to a confining “Cornell” potential $A_0 = -\frac{\varepsilon_F f_0}{\sqrt{2}} r + \frac{Q}{\sqrt{4\pi}\, r}$. When $\Lambda_{\rm eff} = 0$, $A(r) \to 1 - \sqrt{8\pi}|Q|f_0$ for $r \to \infty$ (“hedgehog” **non-flat-spacetime** asymptotics).

Three distinct types of static solutions of “tubelike” LCBR type with space-time geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is some 2-dim manifold ((anti-)de Sitter, Rindler):

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad , \quad -\infty < \eta < \infty \, , \qquad (9)$$

$$F_{0\eta} = c_F = \mathrm{const} \quad , \quad \frac{1}{r_0^2} = 4\pi c_F^2 + \Lambda_0 \; (= \mathrm{const}) \; . \qquad (10)$$

(i) $AdS_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$:

$$A(\eta) = 4\pi\left[c_F^2 - \sqrt{2} f_0 |c_F| - \frac{\Lambda_0}{4\pi}\right]\eta^2 \quad (\eta - \text{Poincare patch coord}) \; , \qquad (11)$$

provided either $|c_F| > \frac{f_0}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\right)$ for $\Lambda_0 \geq -2\pi f_0^2$ or $|c_F| > \sqrt{\frac{1}{4\pi}|\Lambda_0|}$ for $\Lambda_0 < 0 \; , \; |\Lambda_0| > 2\pi f_0^2$.

Generalized Levi-Civita-Bertotti-Robinson Space-Times

(ii) $Rind_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$, where $Rind_2$ is the flat 2-dim Rindler spacetime with:

$$A(\eta) = \eta \;\; \text{for } 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \;\; \text{for } -\infty < \eta < 0 \qquad (12)$$

provided $|c_F| = \frac{f_0}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\right)$ for $\Lambda_0 > -2\pi f_0^2$.

(iii) $dS_2 \times S^2$ with weak const vacuum electric field $|F_{0\eta}| = |c_F|$, where dS_2 is the 2-dim de Sitter space with:

$$A(\eta) = 1 - 4\pi\left[\sqrt{2} f_0 |c_F| - c_F^2 + \frac{\Lambda_0}{4\pi}\right]) \, \eta^2 \;, \qquad (13)$$

when $|c_F| < \frac{f_0}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\right)$ for $\Lambda_0 > -2\pi f_0^2$. Note that dS_2 has *two horizons* at $\eta = \pm\eta_0 \equiv \pm\left[4\pi\left(\sqrt{2} f_0 |c_F| - c_F^2\right) + \Lambda_0\right]^{-\frac{1}{2}}$.

Lagrangian Formulation of Lightlike Brane Dynamics

In what follows we will consider bulk Einstein/non-linear gauge field system (2) self-consistently coupled to $N \geq 1$ (distantly separated) charged codimension-one *lightlike* p-brane (*LL-brane*) sources (here $p = 2$).

World-volume *LL-brane* action in Polyakov-type formulation:

$$S_{\rm LL}[q] = -\frac{1}{2}\int d^{p+1}\sigma\, T b_0^{\frac{p-1}{2}} \sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - b_0(p-1)\right] \;, \quad (1$$

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu}\partial_b X^\nu - \frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) \quad , \quad \mathcal{A}_a \equiv \partial_a X^\mu A_\mu \; . \;(1$$

Here and below the following notations are used:

- γ_{ab} is the *intrinsic* world-volume Riemannian metric; $g_{ab} = \partial_a X^\mu G_{\mu\nu}(X)\partial_b X^\nu$ is the *induced* metric on the world-volume, which becomes *singular* on-shell (manifestation of the lightlike nature); b_0 is world-volume “CC”.

- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk space-time;
- u is auxiliary world-volume scalar field defining the lightlike direction of the induced metric;
- T is *dynamical (variable)* brane tension;
- q – the coupling to bulk spacetime gauge field $\mathcal{A}_\mu$ is *LL-brane* surface charge density.

The on-shell singularity of the induced metric g_{ab} , *i.e.*, the lightlike property, directly follows from the eqs. of motion:

$$g_{ab}\left(\bar{g}^{bc}(\partial_c u + q\mathcal{A}_c)\right) = 0\;. \tag{16}$$

Gravity/Nonlinear Gauge Field Coupled to LL-Branes

Full action of self-consistently coupled bulk Einstein/non-linear gauge field/*LL-brane* ($L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2}$):

$$S = \int d^4x \sqrt{-G}\Big[\frac{R(G) - 2\Lambda_0}{16\pi} + L(F^2)\Big] + \sum_{k=1}^{N} S_{\rm LL}[q^{(k)}] \; , \tag{17}$$

where the superscript (k) indicates the k-th *LL-brane*.

The corresponding equations of motion are as follows:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda_0 G_{\mu\nu} = 8\pi\Big[T^{(F)}_{\mu\nu} + \sum_{k=1}^{N} T^{(k)}_{\mu\nu}\Big] \; , \tag{18}$$

$$\partial_\nu\Big[\sqrt{-G}\Big(1 - \frac{f_0}{\sqrt{-F^2}}\Big)F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda}\Big] + \sum_{k=1}^{N} j^{\mu}_{(k)} = 0 \; . \tag{19}$$

Gravity/Nonlinear Gauge Field Coupled to LL-Branes

The energy-momentum tensor and the charge current density of k-th *LL-brane* are straightforwardly derived from the pertinent *LL-brane* world-volume action (14):

$$T^{\mu\nu}_{(k)} = - \int d^3\sigma \, \frac{\delta^{(4)}\Big(x - X_{(k)}(\sigma)\Big)}{\sqrt{-G}} \, T^{(k)} \sqrt{|\bar{g}_{(k)}|} \bar{g}^{ab}_{(k)} \partial_a X^{\mu}_{(k)} \partial_b X^{\nu}_{(k)} \; , \quad (20)$$

$$j^{\mu}_{(k)} = -q^{(k)} \int d^3\sigma \, \delta^{(4)}\Big(x - X_{(k)}(\sigma)\Big) \sqrt{|\bar{g}_{(k)}|} \bar{g}^{ab}_{(k)} \partial_a X^{\mu}_{(k)} \frac{\partial_b u^{(k)} + q^{(k)} \mathcal{A}^{(k)}_b}{T^{(k)}} \; . \quad (21)$$

Gravity/Nonlinear Gauge Field Coupled to LL-Brane

"Thin-shell" wormhole solutions of static "spherically-symmetric" type (in Eddington-Finkelstein coordinates $dt = dv - \frac{d\eta}{A(\eta)}$):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \quad , \quad F_{v\eta} = F_{v\eta}(\eta) \; , \quad (22)$$

$$-\infty < \eta < \infty \quad , \quad A(\eta_0^{(k)}) = 0 \;\; \text{for} \;\; \eta_0^{(1)} < \ldots < \eta_0^{(N)} \; . \quad (23)$$

(i) Take "vacuum" solutions of (18)–(19) (without delta-function *LL-brane* terms) in each space-time region $\left(-\infty < \eta < \eta_0^{(1)}\right), \ldots,$ $\left(\eta_0^{(N)} < \eta < \infty\right)$ with common horizon(s) at $\eta = \eta_0^{(k)}$ ($k = 1, \ldots, N$).

(ii) Each k-th *LL-brane* automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ – intrinsic property of *LL-brane* dynamics.

(iii) Match discontinuities of the derivatives of the metric and the gauge field strength across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the *LL-brane* stress-energy tensor and charge current density (20)–(21).

Charge-“Hiding” Wormhole

First we will construct “one-throat” wormhole solutions to (17) with the charged *LL-brane* occupying the wormhole “throat”, which connects (i) a non-compact “universe” with Reissner-Nordström-(anti)-de-Sitter-type geometry (where the cosmological constant is partially or entirely *dynamically* generated) to (ii) a compactified (“tubelike”) “universe” of (generalized) Levi-Civita-Bertotti-Robinson type with geometry $AdS_2 \times S^2$ or $Rind_2 \times S^2$.

The whole electric flux produced by the charged *LL-brane* at the wormhole “throat” is pushed into the “tubelike” “universe”. As a result, the non-compact “universe” becomes electrically neutral with Schwarzschild-(anti-)de-Sitter or purely Schwarzschild geometry. Therefore, an external observer in the non-compact “universe” detects a **genuinely charged** matter source (the charged *LL-brane*) as **electrically neutral**.

Charge-"Hiding" Wormhole ("Tubelike" "Left Universe")

Explicit form $ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$ for the metric and the nonlinear gauge theory's electric field $F_{v\eta}(\eta)$:

- "Left universe" of Levi-Civita-Bertotti-Robinson ("tubelike") type with geometry $AdS_2 \times S^2$ for $\eta < 0$:

$$A(\eta) = 4\pi\left(c_F^2 - \sqrt{2}f_0|c_F| - \frac{\Lambda_0}{4\pi}\right)\eta^2 \quad , \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0} \; , \quad (24$$

$$|F_{v\eta}| \equiv |\vec{E}| = |c_F| > \frac{f}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\right) \quad \text{for } \Lambda_0 > -2\pi f_0^2 \; ,$$

$$\text{or} \quad |F_{v\eta}| \equiv |\vec{E}| = |c_F| > \sqrt{\frac{1}{4\pi}|\Lambda_0|} \quad \text{for } \Lambda_0 < 0 \; , \; |\Lambda_0| > 2\pi f_0^2 \; .$$

Charge-“Hiding” Wormhole (Non-Compact “Right Universe”)

Explicit form $ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$ for the metric and the nonlinear gauge theory’s electric field $F_{v\eta}(\eta)$:

- Non-compact “right universe” for $\eta > 0$ comprising the exterior region of RN-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon r_0 when $\Lambda_0 > -2\pi f_0^2$ (in particular, when $\Lambda_0 = 0$), or the exterior region of RN-*anti*-de-Sitter-type black hole beyond the outer (Schwarzschild-type) horizon r_0 in the case $\Lambda_0 < 0$ and $|\Lambda_0| > 2\pi f_0^2$, or the exterior region of RN-“hedgehog” black hole for $|\Lambda_0| = 2\pi f_0^2$ (note: $A(\eta) \equiv A_{\mathrm{RN}-((\mathrm{A})\mathrm{dS})}(r_0+\eta)$):

$$A(\eta) = 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0+\eta} + \frac{Q^2}{(r_0+\eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0+\eta)^2 \; , \quad (2$$

$$C(\eta) = (r_0+\eta)^2 \quad , \quad |F_{v\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0+\eta)^2} \; .$$

Charge “Hiding” Wormhole

The matching relations for the discontinuities of the metric and gauge field components across the *LL-brane* world-volume occupying the wormhole “throat” (which are here derived self-consistently from a well-defined world-volume Lagrangian action principle for the *LL-brane*) determine all parameters of the wormhole solutions as functions of q (the *LL-brane* charge) and f_0 (coupling constant of $\sqrt{-F^2}$):

$$Q = 0 \quad , \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}} \, , \tag{26}$$

as well as the allowed range for the “bare” CC:

$$-4\pi\left(|q| + \frac{f_0}{\sqrt{2}}\right)^2 < \Lambda_0 < 4\pi\left(q^2 - \frac{f_0^2}{2}\right) \, , \tag{27}$$

in particular, Λ_0 could be zero.

Charge "Hiding" Wormhole

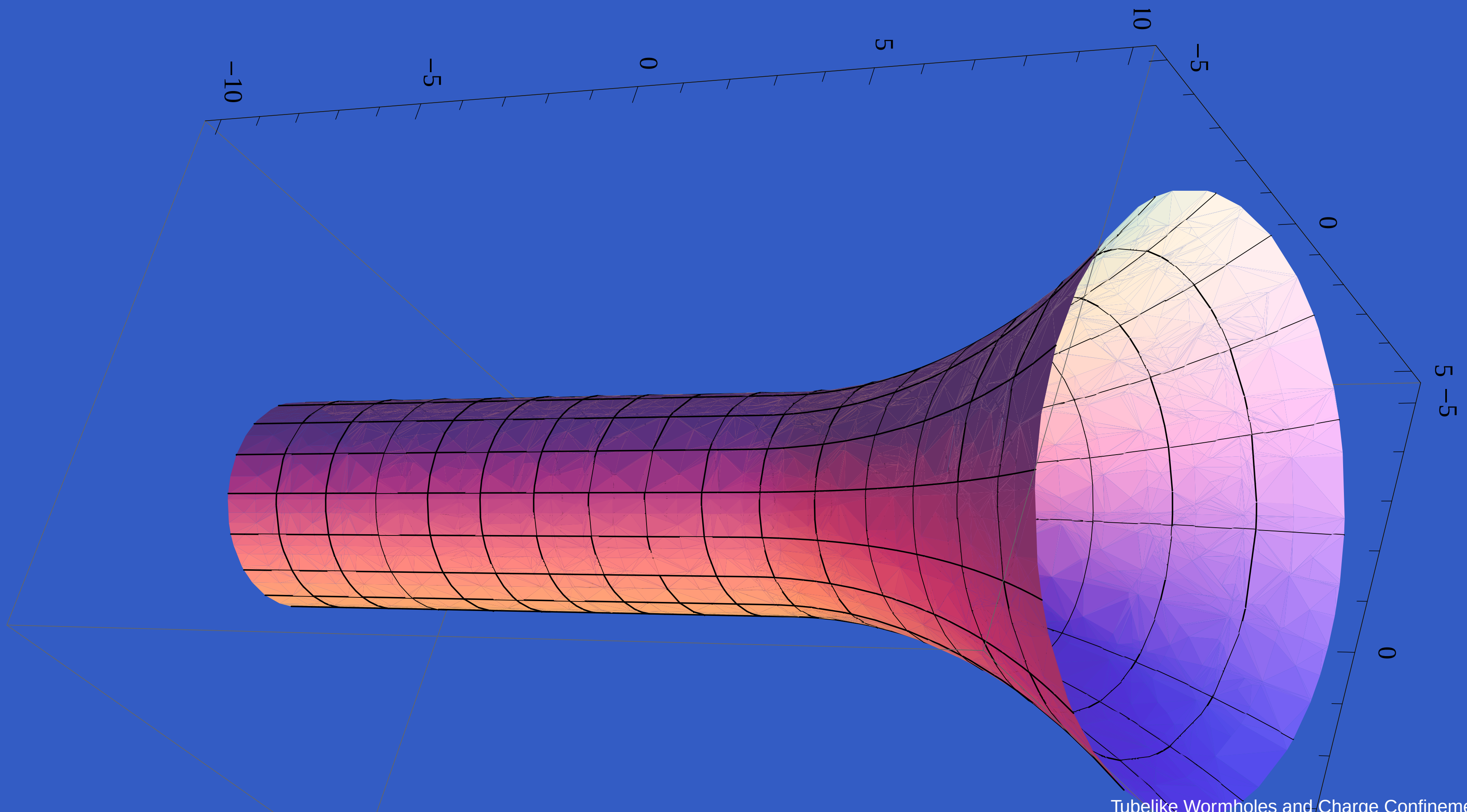
The relations (26) ($Q = 0\,,\ |c_F| = |q| + \frac{f_0}{\sqrt{2}}$; recall $|F_{v\eta}| \equiv |\vec{E}| = |c_F|$ in the "tubelike" "left universe") have profound consequences:

(A) The non-compact "right universe" becomes exterior region of electrically neutral Schwarzschild-(*anti*-)de-Sitter or purely Schwarzschild black hole beyond the Schwarzschild horizon carrying a vacuum constant radial electric field $|F_{v\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}}$.

(B) Recalling that the dielectric displacement field is $\vec{D} = \left(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\right)\vec{E}$, we find from the second rel.(26) that the whole flux produced by the charged *LL-brane* flows only into the "tubelike" "left universe" (since $\vec{D} = 0$ in the non-compact "right universe"). This is a novel property of **hiding electric charge through a wormhole** connecting non-compact to a "tubelike" universe from external observer in the non-compact "universe".

Visualizing Charge-“Hiding” Wormhole

Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of **charge-“hiding”** wormhole geometry: the whole electric flux produced by the charged *LL-brane* at the “throat” is expelled into the left infinitely long cylindric tube.



Charge Confining Wormhole (Non-Compact “Left Universe”)

There exist more interesting “two-throat” wormhole solution exhibiting **QCD-like charge confinement** effect – obtained from a self-consistent coupling of the gravity/nonlinear-gauge-field system (2) with two identical *oppositely charged LL-branes*. The total “two-throat” wormhole space-time manifold is made of:

(i) “Left-most” non-compact “universe” comprising the exterior region of RN-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the “radial-like” η-coordinate interval $-\infty < \eta < -\eta_0 \equiv -\left[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\right]^{-\frac{1}{2}}$, where:

$$A(\eta) = A_{\mathrm{RNdS}}(r_0 - \eta_0 - \eta) =$$

$$1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0 - \eta_0 - \eta} + \frac{Q^2}{(r_0 - \eta_0 - \eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0 - \eta_0 - \eta)^2$$

$$C(\eta) = (r_0 - \eta_0 - \eta)^2 \quad , \quad |F_{v\eta}(\eta)| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0 - \eta_0 - \eta)^2} \, .$$

Charge Confining Wormhole (“Tubelike” “Middle Universe”)

(ii) “Middle” “tube-like” “universe” of Levi-Civita-Bertotti-Robinson type with geometry $dS_2 \times S^2$ comprising the finite extent (w.r.t. η-coordinate) region between the two horizons of dS_2 at $\eta = \pm\eta_0$:

$$-\eta_0 < \eta < \eta_0 \equiv \left[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\right]^{-\frac{1}{2}} , \qquad (28)$$

where the metric coefficients and electric field are:

$$A(\eta) = 1 - \left[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\right]\eta^2 \;\; , \quad A(\pm\eta_0) = 0 \; ,$$

$$C(\eta) = r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0} \quad , \quad |F_{v\eta}| \equiv |\vec{E}| = |c_F| < \frac{f}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f_0^2}}\right)$$

with $\Lambda_0 > -2\pi f_0^2$;

Charge Confining Wormhole (Non-Compact "Right Universe")

(iii) "Right-most" non-compact "universe" comprising the exterior region of RN-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval $\eta_0 < \eta < \infty$ (η_0 as in (28)), where:

$$A(\eta) = A_{\mathrm{RNdS}}(r_0 + \eta - \eta_0)$$

$$= 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0 + \eta - \eta_0} + \frac{Q^2}{(r_0 + \eta - \eta_0)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0 + \eta - \eta_0$$

$$C(\eta) = (r_0 + \eta - \eta_0)^2 \quad , \quad |F_{v\eta}(\eta)| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0 + \eta - \eta_0}$$

As dictated by the *LL-brane* dynamics each of the two *LL-branes* locates itself on one of the two common horizons at $\eta = \pm\eta_0$ between "left" and "middle", and between "middle" and "right" "universes", respectively.

Charge Confining Wormhole (Non-Compact “Right Universe”)

The matching relations for the discontinuities of the metric and gauge field components across the each of the two *LL-brane* world-volumes determine all parameters of the wormhole solutions as functions of $\pm q$ (the opposite *LL-brane* charges) and f_0 (coupling constant of $\sqrt{-F^2}$). Most importantly we obtain:

$$Q = 0 \quad , \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}} \; , \tag{29}$$

and bare cosmological constant must be in the interval:

$$\Lambda_0 \leq 0 \quad , \quad |\Lambda_0| < 2\pi(f_0^2 - 2q^2) \quad \to \quad |q| < \frac{f_0}{\sqrt{2}} \; , \tag{30}$$

in particular, Λ_0 could be zero.

Charge-Confining Wormhole

Similarly to the charge-"hiding" case, rel.(29) $Q = 0$ and $|c_F| = |q| + \frac{f_0}{\sqrt{2}}$, which means:

$$|\vec{E}|_{\text{middle universe}} = |q| + |\vec{E}|_{\text{left/right universe}} \;,$$

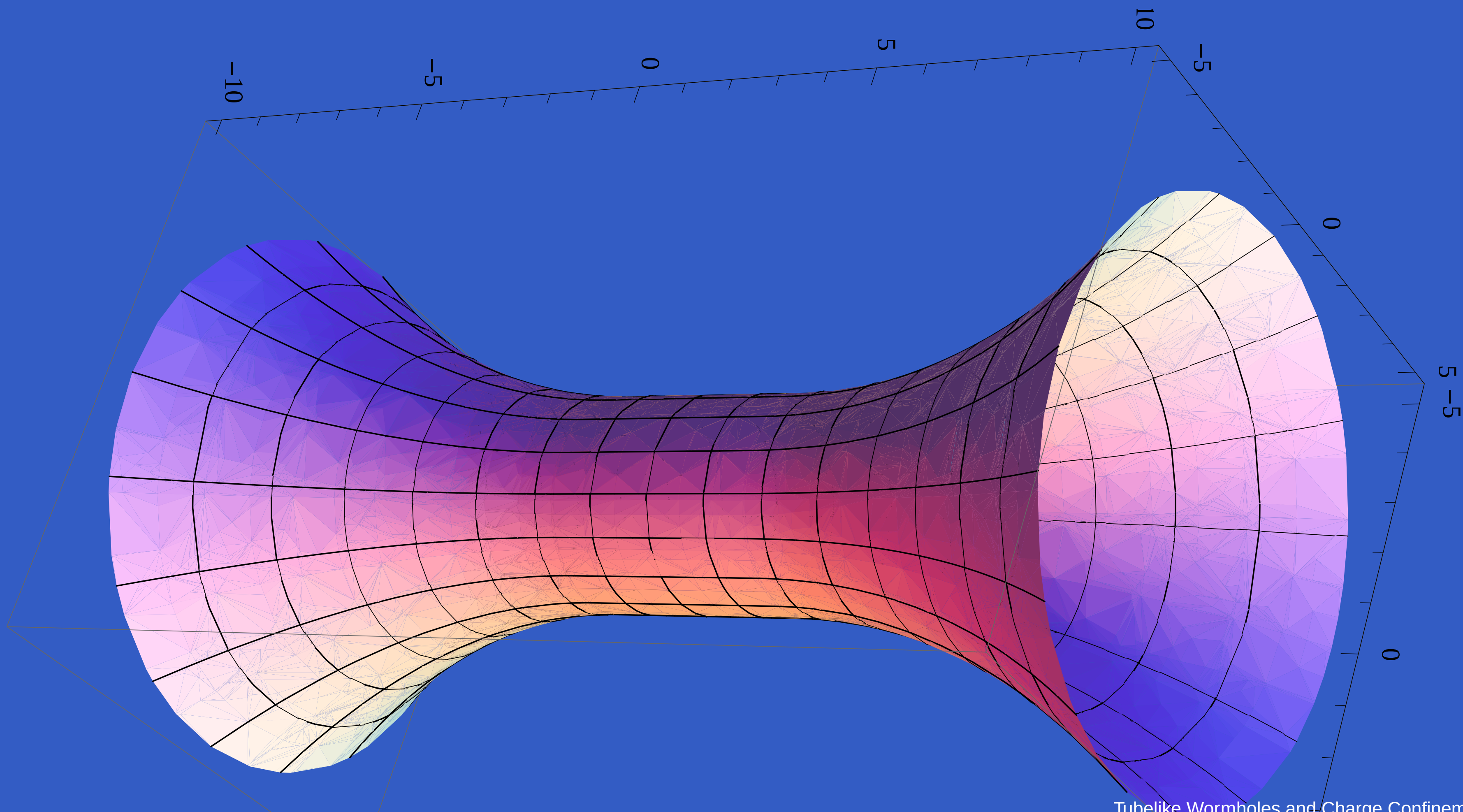
have profound consequences:

- The "left-most" and "right-most" non-compact "universes" become two identical copies of the *electrically neutral* exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. They both carry a constant vacuum radial electric field with magnitude $|\vec{E}| = \frac{f_0}{\sqrt{2}}$ pointing inbound towards the horizon in one of these "universes" and pointing outbound w.r.t. the horizon in the second "universe". The corresponding electric displacement field $\vec{D} = 0$, so there is *no* electric flux there (recall $\vec{D} = \left(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\right) \vec{E}$).

Charge-Confining Wormhole

- The whole electric flux produced by the two charged *LL-branes* with opposite charges $\pm q$ at the boundaries of the above non-compact "universes" is *confined* within the "tube-like" middle "universe" of Levi-Civitta-Robinson-Bertotti type with geometry $dS_2 \times S^2$, where the constant electric field is $|\vec{E}| = \frac{f_0}{\sqrt{2}} + |q|$ with associated non-zero electric displacement field $|\vec{D}| = |q|$. This is **QCD-like confinement**.

Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of **charge-confining** wormhole geometry: the whole electric flux produced by the two oppositely charged *LL-branes* is confined within the middle finite-extent cylindric tube between the “throats”.



Dynamical Couplings & Confinement/Deconfinement from R^2-Gravity

Consider now coupling of $f(R) = R + \alpha R^2$ gravity (possibly with a bare cosmological constant Λ_0) to a “dilaton” ϕ and the nonlinear gauge field system containing $\sqrt{-F^2}$:

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{16\pi} \Big(f\big(R(g,\Gamma)\big) - 2\Lambda_0 \Big) + L(F^2(g)) + L_D(\phi, g) \Big] , \quad (31)$$

$$f\big(R(g,\Gamma)\big) = R(g,\Gamma) + \alpha R^2(g,\Gamma) \quad , \quad R(g,\Gamma) = R_{\mu\nu}(\Gamma) g^{\mu\nu} , \quad (32)$$

$$L(F^2(g)) = -\frac{1}{4e^2} F^2(g) - \frac{f_0}{2} \sqrt{-F^2(g)} , \quad (33)$$

$$F^2(g) \equiv F_{\kappa\lambda} F_{\mu\nu} g^{\kappa\mu} g^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (34)$$

$$L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) . \quad (35)$$

$R_{\mu\nu}(\Gamma)$ is the Ricci curvature in the first order (Palatini) formalism, *i.e.*, the space-time metric $g_{\mu\nu}$ and the affine connection $\Gamma^\mu_{\nu\lambda}$ are *a priori* independent variables.

The equations of motion resulting from the action (31) read:

$$R_{\mu\nu}(\Gamma) = \frac{1}{f'_R}\left[\kappa^2 T_{\mu\nu} + \frac{1}{2} f\big(R(g,\Gamma)\big) g_{\mu\nu}\right] \quad , \quad f'_R \equiv \frac{df(R)}{dR} = 1 + 2\alpha R(g,\Gamma) \; , \tag{36}$$

$$\nabla_\lambda \left(\sqrt{-g} f'_R g^{\mu\nu}\right) = 0 \; , \tag{37}$$

$$\partial_\nu \left(\sqrt{-g}\left[1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}}\right] F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda}\right) = 0 \; . \tag{38}$$

The total energy-momentum tensor is given by:

$$T_{\mu\nu} = \left[L(F^2(g)) + L_D(\phi, g) - \frac{1}{\kappa^2}\Lambda_0\right] g_{\mu\nu} + \left(1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}}\right) F_{\mu\kappa} F_{\nu\lambda} g^{\kappa\lambda} + \partial_\mu \phi \partial_\nu \phi \; . \tag{39}$$

Dynamical Couplings & Confinement/Deconfinement from R^2-Gravity

Eq.(37) leads to the relation $\nabla_\lambda \left(f'_R g_{\mu\nu}\right) = 0$ and thus it implies transition to the "physical" Einstein-frame metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$:

$$g_{\mu\nu} = \frac{1}{f'_R} h_{\mu\nu} \quad , \quad \Gamma^\mu_{\nu\lambda} = \frac{1}{2} h^{\mu\kappa} \left(\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}\right) \; . \qquad (40)$$

Using (40) the R^2-gravity eqs. of motion (36) can be rewritten in the form of *standard* Einstein equations:

$$R^\mu_\nu(h) = 8\pi \left(T_{\mathrm{eff}}{}^\mu_\nu(h) - \frac{1}{2}\delta^\mu_\nu T_{\mathrm{eff}}{}^\lambda_\lambda(h) \right) \qquad (41)$$

with effective energy-momentum tensor of the following form:

$$T_{\mathrm{eff}\,\mu\nu}(h) = h_{\mu\nu} L_{\mathrm{eff}}(h) - 2\frac{\partial L_{\mathrm{eff}}}{\partial h^{\mu\nu}} \; . \qquad (42)$$

The effective Einstein-frame matter lagrangian reads (here $X(\phi, h) \equiv -\frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_n \phi$, to be ignored in the sequel):

$$L_{\text{eff}}(h) = -\frac{1}{4e_{\text{eff}}^2(\phi)} F^2(h) - \frac{1}{2} f_{\text{eff}}(\phi) \sqrt{-F^2(h)} + \frac{X(\phi,h)\big(1 + 16\pi\alpha X(\phi,h)\big) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \tag{43}$$

with the following dynamical ϕ-dependent couplings:

$$\frac{1}{e_{\text{eff}}^2(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} , \tag{44}$$

$$f_{\text{eff}}(\phi) = f_0 \frac{1 + 32\pi\alpha X(\phi,h)}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} . \tag{45}$$

Thus, all eqs. of motion of the original R^2-gravity system (31)–(35) can be equivalently derived from the following Einstein/nonlinear-gauge-field/dilaton action:

$$S_{\rm eff} = \int d^4x\sqrt{-h}\Big[\frac{R(h)}{16\pi} + L_{\rm eff}(h)\Big] \; , \qquad (46)$$

where $R(h)$ is the standard Ricci scalar of the metric $h_{\mu\nu}$ and $L_{\rm eff}(h)$ is as in (43).

Important observation. Even if ordinary kinetic Maxwell term $-\frac{1}{4}F^2$ is absent in the original system ($e^2 \to \infty$ in (33)), such term is nevertheless *dynamically generated* in the Einstein-frame action (43)–(46) – *combined effect* of αR^2 and $-\frac{f_0}{2}\sqrt{-F^2}$:

$$S_{\rm maxwell} = -4\pi\alpha f_0^2 \int d^4x\sqrt{-h}\frac{F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu}}{1+8\alpha\left(8\pi V(\phi)+\Lambda_0\right)} \; . \qquad (47)$$

Dynamical Couplings & Confinement/Deconfinement from R^2-Gravity

In what follows we consider constant "dilaton" ϕ extremizing the effective Lagrangian (43):

$$L_{\rm eff} = -\frac{1}{4e^2_{\rm eff}(\phi)} F^2(h) - \frac{1}{2} f_{\rm eff}(\phi)\sqrt{-F^2(h)} - V_{\rm eff}(\phi) \; , \tag{48}$$

$$V_{\rm eff}(\phi) = \frac{V(\phi) + \frac{\Lambda_0}{8\pi}}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; , \quad f_{\rm eff}(\phi) = \frac{f_0}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; , \tag{49}$$

$$\frac{1}{e^2_{\rm eff}(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; . \tag{50}$$

The dynamical couplings and effective potential are extremized **simultaneously** – explicit realization of "least coupling principle" of Damour-Polyakov:

$$\frac{\partial f_{\rm eff}}{\partial\phi} = -64\pi\alpha f_0 \frac{\partial V_{\rm eff}}{\partial\phi} \; , \quad \frac{\partial}{\partial\phi}\frac{1}{e^2_{\rm eff}} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\rm eff}}{\partial\phi} \quad \to \frac{\partial L_{\rm eff}}{\partial\phi} \sim \frac{\partial V_{\rm eff}}{\partial\phi} \; . \tag{51}$$

Therefore at the extremum of $L_{\rm eff}$ (48) ϕ must satisfy:

$$\frac{\partial V_{\rm eff}}{\partial \phi} = \frac{V'(\phi)}{\left[1 + 8\alpha\left(\kappa^2 V(\phi) + \Lambda_0\right)\right]^2} = 0\ . \qquad (52)$$

There are two generic cases:

(a) **Confining phase**: Eq.(52) is satisfied for some finite-value ϕ_0 extremizing the original potential $V(\phi)$: $V'(\phi_0) = 0$.

(b) **Deconfiment phase**: For polynomial or exponentially growing original $V(\phi)$, so that $V(\phi) \to \infty$ when $\phi \to \infty$, we have:

$$\frac{\partial V_{\rm eff}}{\partial \phi} \to 0 \quad , \quad V_{\rm eff}(\phi) \to \frac{1}{64\pi\alpha} = \mathrm{const} \quad \mathrm{when} \quad \phi \to \infty\ , \qquad (53)$$

i.e., for sufficiently large values of ϕ we find a “flat region” in $V_{\rm eff}$. This “flat region” triggers a **transition from confining to deconfinement dynamics**.

Namely, in the “flat-region” case we have:

$$f_{\rm eff} \to 0 \quad , \quad e^2_{\rm eff} \to e^2 \tag{54}$$

and the effective gauge field Lagrangian (48) reduces to the ordinary *non-confining* one (the “square-root” term $\sqrt{-F^2}$ vanishes):

$$L^{(0)}_{\rm eff} = -\frac{1}{4e^2}F^2(h) - \frac{1}{64\pi\alpha} \tag{55}$$

with an *induced* cosmological constant $\Lambda_{\rm eff} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0.

Static spherically symmetric solutions of R^2-gravity

Within the physical "Einstein"-frame in the confining phase:

(A) Reissner-Nordström-(*anti*-)de-Sitter type black holes, in particular, non-standard Reissner-Nordström type with non-flat "hedgehog" asymptotics, where now:

$$\Lambda_{\rm eff}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha\left(\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2\right)} \; , \quad (56)$$

$$|\vec{E}_{\rm vac}| = \left(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi_0) + \Lambda_0\right)}\right)^{-1} \frac{f_0/\sqrt{2}}{1 + 8\alpha\left(8\pi V(\phi_0) + \Lambda_0\right)} \; . \quad (57)$$

(B) Levi-Civitta-Bertotti-Robinson type "tubelike" space-times with geometries $AdS_2 \times S^2$, $Rind_2 \times S^2$ and $dS_2 \times S^2$ where now (using short-hand $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1 + 8\alpha\Lambda(\phi_0)}\left[\left(1 + 8\alpha\left(\Lambda(\phi_0) + 2\pi f_0^2\right)\right)\vec{E}^2 + \frac{1}{4\pi}\Lambda(\phi_0)\right] \; . \quad (58)$$

Conclusions

Inclusion of the non-standard nonlinear “square-root” gauge field term – explicit realization of the old “classic” idea of ‘t Hooft about the nature of low-energy confinement dynamics.

Coupling of nonlinear gauge theory containing $\sqrt{-F^2}$ to gravity (Einstein or $f(R) = R + \alpha R^2$ plus scalar “dilaton”) leads to a variety of remarkable effects:

- Dynamical effective gauge couplings and dynamical induced cosmological constant;
- New non-standard black hole solutions of RN-(*anti*-)de-Sitter type carrying an additional constant vacuum electric field, in particular, non-standard RN type black holes with asymptotically non-flat “hedgehog” behaviour;
- “Cornell”-type confining potential in charged test particle dynamics;

Conclusions

- Coupling to a charged lightlike brane produces a charge-“hiding” wormhole, where a genuinely charged matter source is detected as electrically neutral by an external observer;
- Coupling to two oppositely charged lightlike brane sources produces a two-“throat” wormhole displaying a genuine QCD-like charge confinement.
- When coupled to $f(R) = R + \alpha R^2$ gravity plus scalar “dilaton”, the $\sqrt{-F^2}$ term triggers a transition from confining to deconfinement phase. Standard Maxwell kinetic term for the gauge field is dynamically generated even when absent in the original “bare” theory. The above are cumulative effects produced by the **simultaneous** presence of αR^2 and $\sqrt{-F^2}$ terms.