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AdS-inspired noncommutative gravity

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M. Dimitrijević and V. Radovanović, arXiv: 1207.4675[hep-th]

P. Aschieri, L. Castellani and M. Dimitrijević, arXiv: 1207.4346[hep-th]

Overview

Motivation

Why Noncommutativity? Physics at small distances (high energies) not very well understood.

Original motivation: to solve the problem of divergences in QFT.

Recent motivation: quantum Hall effect, appears in string theory, playground for Quantum theory of gravity, new effects in QFT, . . . [Douglas, Nekrasov '01; Szabo '01, '06,; Castellani '00;. . .]

Why noncommutative NC gravity?

No renormalizable gravity theory (yet), modified gravity theory could explain the problems such as Dark matter and Dark energy,. . .

How to do NC gravity? Not clear, different approaches: twisted diffeomorphism symmetry [Aschieri, Blohmann, Dimitrijević, Meyer, Schupp, Wess '05, '06], NC Lorentz gauge theory [Chamseddine '01,'04, Cardela, Zanon '03, Aschieri, Castellani '09,'12],. . .

Reminder I: Commutative AdS gauge theory

The ispometry group of AdS space-time is the $SO(2,3)$ group. It is generated by M_{AB}, $A, B = 0, \dots, 3, 5$ and

$$[M_{AB}, M_{CD}] = i(\eta_{AD}M_{BC} + \eta_{BC}M_{AD} - \eta_{AC}M_{BD} - \eta_{BD}M_{AC}), \quad (1)$$

with the metric tensor $\eta_{AB} = \mathrm{diag}(+,-,-,-,+)$.

Various representations of (1); especially useful representation is given in terms of gamma matrices:

$$M_{AB} = \frac{i}{2}[\Gamma_A, \Gamma_B] \ , \quad \{\Gamma_A, \Gamma_B\} = 2\eta_{AB}. \quad (2)$$

Here $\Gamma_A = (i\gamma_a\gamma_5, \gamma_5)$, $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $a = 0, \dots, 3$.

Note that

$$M_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\sigma_{ab}, \quad M_{5a} = \frac{i}{2}\gamma_a. \quad (3)$$

If we introduce momenta $P_a = \frac{1}{l} M_{a5}$, where the constant l has dimension of length, the AdS algebra (1) becomes

$$\begin{aligned}
[M_{ab}, M_{cd}] &= i(\eta_{ad} M_{bc} + \eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac}), \\
[M_{ab}, P_c] &= i(\eta_{bc} P_a - \eta_{ac} P_b), \\
[P_a, P_b] &= -i \frac{1}{l^2} M_{ab}. \qquad (4)
\end{aligned}$$

In the limit $l \to \infty$ AdS algebra reduces usual Poincaré algebra in $4D$ spacetime, Inonü-Wigner contraction.

This can be used to generate General Relativity (GR) from AdS gauge theory on 4 dimensional Minkowski space-time.

Gauge parameter

$$\epsilon = \frac{1}{2}\epsilon^{AB} M_{AB} = \frac{1}{4}\epsilon^{ab}\sigma_{ab} + \frac{1}{2}\epsilon^{a5}\gamma_a. \tag{5}$$

Gauge field

$$\omega_\mu = \frac{1}{2}\omega_\mu^{AB} M_{AB} = \frac{1}{4}\omega^{ab}\sigma_{ab} + \frac{1}{2}\omega_\mu^{a5}\gamma_a. \tag{6}$$

Infinitesimal gauge transformations are given by

$$\delta_\epsilon \omega_\mu = \partial_\mu \epsilon + i[\epsilon, \omega_\mu]. \tag{7}$$

Field strength tensor

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu - i[\omega_\mu, \omega_\nu] = \frac{1}{2}F_{\mu\nu}^{AB} M_{AB} \\ &= \frac{1}{4}\Big(R_{\mu\nu}^{ab} - \frac{1}{l^2}(e_\mu^a e_\nu^b - e_\mu^b e_\nu^a)\Big)\sigma_{ab} + \frac{1}{2}F_{\mu\nu}^{a5}\gamma_a, \end{aligned} \tag{8}$$

with

$$\begin{aligned} R_{\mu\nu}^{ab} &= \partial_\mu\omega_\nu^{ab} - \partial_\nu\omega_\mu^{ab} + \omega_{\mu c}^{a}\omega_\nu^{cb} - \omega_{\mu c}^{b}\omega_\nu^{ca}, \\ F_{\mu\nu}^{a5} &= \frac{1}{l}\Big(D_\mu e_\nu{}^a - D_\nu e_\mu{}^a\Big), \quad D_\mu e_\nu{}^a = \partial_\mu e_\nu{}^a + \omega_{\mu b}^{a} e_\nu{}^b. \end{aligned}$$

The action is given by [Stelle, West '80, MacDowell, Mansouri '77]

$$S = \frac{il}{64\pi G_N} \mathrm{Tr} \int \mathrm{d}^4 x \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \Phi + \lambda \int \mathrm{d}^4 x \left(\frac{1}{4} \mathrm{Tr}\, \Phi^2 - l^2 \right), \quad (9)$$

with $\Phi = \Phi^A \Gamma_A$ and $\delta_\epsilon \Phi = i[\epsilon, \Phi]$; G_N is the Newton's gravitational constant and λ is the Lagrange multiplier.

The constraint on Φ is $\mathrm{Tr}\, \Phi^2 = l^2$. Choosing $\Phi^a = 0$ and $\Phi^5 = l$ breaks $SO(2,3)$ symmetry down to $SO(1,3)$ gauge symmetry. The action after SSB is

$$\begin{aligned} S &= -\frac{il^2}{64\pi G_N} \int \mathrm{d}^4 x \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \left(F_{\mu\nu} F_{\rho\sigma} \gamma_5 \right) \quad (10) \\ &= \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \left(\frac{l^2}{16} \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} R_{\mu\nu}^{\ \ ab} R_{\rho\sigma}^{\ \ cd} + \sqrt{-g} R - 2\sqrt{-g} \Lambda \right) \end{aligned}$$

and $\Lambda = -3/l^2$, $\sqrt{-g} = \det e_\mu^a$, $R = R_{\mu\nu}^{\ \ ab} e_a^{\ \mu} e_b^{\ \nu}$.

Comments:

-variables in (10) are spin connection ω_μ and vielbeins e_μ. They are independent, 1st order formalsim.

-(10) is invariant under $SO(1,3)$ gauge symmetry, while the diffeomorphism symmetry appears as a consequence of SSB, see [Stelle, West '80].

-varying (10) with respect to ω_μ and vielbeins e_μ gives equations of motion for these fields. The spin connection in not dynamical (the equation of motion is algebraic, the zero-torsion condition) and can be expressed in terms of vielbeins, 2nd order formalism.

-(10) written in the 2nd order formalism has three terms: Gauss-Bonnet topological term, Einstein-Hilbert term and the cosmological constant term.

NC gauge theory via SW map

We work with canonical (θ-constant) noncommutativity and in the $\star$-product approach (representation of the NC algebra of functions on the space of commuting coordinates):

$$f \cdot g \quad \to \quad f \star g = e^{\frac{i}{2}\theta^{\alpha\beta}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}} f(x) g(y)|_{y\to x}$$

$$= f \cdot g + \frac{i}{2}\theta^{\alpha\beta}(\partial_\alpha f)(\partial_\beta g)$$

$$-\frac{1}{8}\theta^{\alpha\beta}\theta^{\kappa\lambda}(\partial_\alpha\partial_\kappa f)(\partial_\beta\partial_\lambda g) + \dots$$

$$\epsilon,\ \Phi,\ \omega_\mu,\ F_{\mu\nu} \quad \to \quad \hat\epsilon,\ \hat\Phi,\ \hat\omega_\mu,\ \hat F_{\mu\nu} = \partial_\mu\hat\omega_\nu - \partial_\nu\hat\omega_\mu - i[\hat\omega_\mu \overset{\star}{,} \hat\omega_\nu]$$

$$\delta_\epsilon\Phi = i[\epsilon, \Phi] \quad \to \quad \delta^\star_\epsilon\hat\Phi = i[\hat\epsilon \overset{\star}{,} \hat\Phi]$$

$$\delta_\epsilon\omega_\mu = \partial_\mu\epsilon + i[\epsilon, \omega_\mu] \quad \to \quad \delta^\star_\epsilon\hat\omega_\mu = \partial_\mu\hat\epsilon + i[\hat\epsilon \overset{\star}{,} \hat\omega_\mu]$$

$$\delta_\epsilon F_{\mu\nu} = i[\epsilon, F_{\mu\nu}] \quad \to \quad \delta^\star_\epsilon\hat F_{\mu\nu} = i[\hat\epsilon \overset{\star}{,} \hat F_{\mu\nu}]$$

$\star$-commutators do not close in the Lie algebra of the gauge group.
Example: $\hat{\epsilon}_{1,2} = \hat{\epsilon}^a_{1,2} T^a$, $[T^a, T^b] = if^{abc} T^c$ and

$$
\begin{aligned}
[\hat{\epsilon}_1 \overset{\star}{,} \hat{\epsilon}_2] &= \hat{\epsilon}_1 \star \hat{\epsilon}_2 - \hat{\epsilon}_2 \star \hat{\epsilon}_1 \\
&= \frac{1}{2}\{\hat{\epsilon}_1{}^a \overset{\star}{,} \hat{\epsilon}_2{}^b\}[T^a, T^b] + \frac{1}{2}[\hat{\epsilon}_1{}^a \overset{\star}{,} \hat{\epsilon}_2^b]\{T^a, T^b\} \\
&\notin \textit{Lie algebra}
\end{aligned}
$$

except for NC $U(N)$ gauge theory. In general, NC gauge parameter and NC gauge fields have to be enveloping algebra-valued:

$$
\begin{aligned}
\hat{\omega}_\mu &= \omega_\mu^{(0)a} T^a + \omega_\mu^{(1)ab} \frac{1}{2}\{T^a, T^b\} \\
&\quad + \omega_\mu^{(2)abc} \frac{1}{3!} \sum_\sigma T^{\sigma(a)} T^{\sigma(b)} T^{\sigma(c)} + \dots
\end{aligned}
$$

New fields $\omega_\mu^{(1)ab}, \omega_\mu^{(2)abc}, \ldots \Rightarrow$ new degrees of freedom, infinitely many!

The main idea of the Seiberg-Witten map: NC gauge transformations are induced by the commutative ones, $\delta_\epsilon \to \delta^\star_\epsilon$. Then:

$$\hat{\epsilon} = \hat{\epsilon}(\epsilon, \omega_\mu), \quad \hat{\omega}_\mu = \hat{\omega}_\mu(\omega_\mu), \quad \hat{\Phi} = \hat{\Phi}(\Phi, \omega_\mu). \tag{11}$$

The consistency relation for gauge transformations

$$(\delta^\star_{\epsilon_1}\delta^\star_{\epsilon_2} - \delta^\star_{\epsilon_2}\delta^\star_{\epsilon_1})\hat{\Phi} = \delta^\star_{-i[\alpha,\beta]}\hat{\Phi}$$

yields the solution for $\hat{\epsilon}(\epsilon, \omega_\mu)$:

$$\hat{\epsilon}(\epsilon, \omega_\mu) = \epsilon^a T^a - \frac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha, \partial_\beta \epsilon\} + \dots \tag{12}$$

NC filed $\hat{\Phi}$ with $\delta^\star_\epsilon \hat{\Phi} = i[\hat{\epsilon} \stackrel{\star}{,} \hat{\Phi}]$:

$$\hat{\Phi} = \Phi^a T^a - \frac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha, \partial_\beta\Phi + D_\beta\Phi\} + \dots, \tag{13}$$

with $D_\beta\Phi = \partial_\beta\Phi - i[\omega_\beta, \Phi]$.

NC gauge field $\hat{\omega}_\mu$ with $\delta^\star_\epsilon \hat{\omega}_\mu = \partial_\mu \hat{\epsilon} + i[\hat{\epsilon} \stackrel{\star}{,} \hat{\omega}_\mu]$:

$$\hat{\omega}_\mu = \omega^a_\mu T^a - \frac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha, \partial_\beta\omega_\mu + F_{\beta\mu}\} + \dots . \tag{14}$$

The main result of the SW map: No new degrees of freedom, NC gauge theory and the corresponding commutative gauge theory have the same number of degrees of freedom! Construction of the higher order solutions is not a problem, [Ulker, Yapiskan '08, Aschieri, Castellani '11].

AdS NC gravity: construction

We are interested in a NC generalization of (9). However, there are problems with the breaking of NC symmetry (work in progress):

$$
\begin{array}{ccc}
SO(2,3)_\star & \longleftrightarrow & SO(2,3) \\
\\
\not{ssb} \downarrow & & \downarrow ssb \\
\\
SO(1,3)_\star & \longleftrightarrow & SO(1,3)
\end{array}
$$

Therefore, we start from the action (10) and construct its NC generalization.

We start from

$$S = -\frac{il^2}{64\pi G_N}\int \mathrm{d}^4x\epsilon^{\mu\nu\rho\sigma}\Big(\mathrm{Tr}\,(\hat{R}_{\mu\nu}\star\hat{R}_{\rho\sigma}\gamma_5) - \frac{2i}{l^2}\mathrm{Tr}\,(\hat{R}_{\mu\nu}\star\hat{E}_\rho\star\hat{E}_\sigma\gamma_5) + \frac{1}{l^4}\mathrm{Tr}\,(\hat{E}_\mu\star\hat{E}_\nu\star\hat{E}_\rho\star\hat{E}_\sigma\gamma_5)\Big], \tag{15}$$

with:

-NC $SO(1,3)_\star$ gauge potential: $\hat{\omega}_\mu = \omega_\mu + \hat{\omega}_\mu^{(1)} + \hat{\omega}_\mu^{(2)} + \dots$

-NC vielbeins: $\hat{E}_\mu = e_\mu + \hat{E}_\mu^{(1)} + \hat{E}_\mu^{(2)} + \dots$

-NC curvature tensor:

$$\begin{aligned}\hat{R}_{\mu\nu} &= \partial_\mu\hat{\omega}_\nu - \partial_\nu\hat{\omega}_\mu - i[\hat{\omega}_\mu \stackrel{\star}{,} \hat{\omega}_\nu] \\ &= R_{\mu\nu} + \hat{R}_{\mu\nu}^{(1)} + \hat{R}_{\mu\nu}^{(2)} + \dots.\end{aligned}$$

The action is invariant under the NC $SO(1,3)_\star$ symmetry (important: the integral is cyclic). The NC fields transform as:

$$\begin{aligned} \delta^\star_\epsilon \hat{\omega}_\mu &= \partial_\mu \hat{\epsilon} + i[\hat{\epsilon} \stackrel{\star}{,} \hat{\omega}_\mu], \\ \delta^\star_\epsilon \hat{R}_{\mu\nu} &= i[\hat{\epsilon} \stackrel{\star}{,} \hat{R}_{\mu\nu}], \\ \delta^\star_\epsilon \hat{E}_\mu &= i[\hat{\epsilon} \stackrel{\star}{,} \hat{E}_\mu]. \end{aligned} \tag{16}$$

The NC fields are valued in the universal enveloping algebra (UEA) of $SO(1,3)$. The $\star$-commutators in (16) only close in UEA.

We use the SW map to expand NC fields in terms of the corresponding commutative fields and calculate $\hat{E}^{(1)}_\mu$, $\hat{E}^{(2)}_\mu$, ...

For NC vielbeins we find

$$\hat{E}^{(n+1)}_\mu = -\frac{1}{4(n+1)}\theta^{\kappa\lambda}\Big(\{\hat{\omega}_\kappa \stackrel{\star}{,} \partial_\lambda \hat{E}_\mu + D_\lambda \hat{E}_\mu\}\Big)^{(n)} \qquad (17)$$

$$\hat{E}_\mu = e_\mu - \frac{1}{4}\theta^{\kappa\lambda}\{\omega_\kappa, \partial_\lambda e_\mu + D_\lambda e_\mu\} + \dots$$

$$\hat{E}_\mu = E^a_\mu \gamma_a + E^{5a}_\mu \gamma_a\gamma_5,$$

with $D_\lambda \hat{E}_\mu = \partial_\lambda \hat{E}_\mu - i[\hat{\omega}_\lambda \stackrel{\star}{,} \hat{E}_\mu]$.

The NC curvature tensor is given by

$$\hat{R}^{(n+1)}_{\mu\nu} = -\frac{1}{4(n+1)}\theta^{\kappa\lambda}\Big(\{\hat{\omega}_\kappa \stackrel{\star}{,} \partial_\lambda \hat{R}_{\mu\nu} + D_\lambda \hat{R}_{\mu\nu}\}\Big)^{(n)} + \frac{1}{2(n+1)}\theta^{\kappa\lambda}\Big(\{\hat{R}_{\mu\kappa}, \stackrel{\star}{,} \hat{R}_{\nu\lambda}\}\Big)^{(n)}$$

$$\hat{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}\theta^{\kappa\lambda}\{\omega_\kappa, \partial_\lambda R_{\mu\nu} + D_\lambda R_{\mu\nu}\} + \frac{1}{2}\theta^{\kappa\lambda}\{R_{\mu\kappa}, R_{\nu\lambda}\} + \qquad (18)$$

$$= \frac{1}{4}\tilde{R}^{ab}_{\mu\nu}\sigma_{ab} + \tilde{R}_{\mu\nu} I + \tilde{R}^5_{\mu\nu}\gamma_5.$$

AdS NC gravity: expansion

Solutions of the SW map we insert into the action (15) and calculate corrections. We find:

$$\begin{aligned} S^{(0)} &= \text{commutative action (10)} \\ S^{(1)} &= 0 \text{ unfortunately!!!} \\ S^{(2)} &= \text{very complicated, not manifestly gauge invariant,} \\ &\quad \text{no explicit results existed until June 2012.} \end{aligned}$$

Shortcut: SW map for composite fields [Aschieri, Castellani, Dimitrijevic, '12].

An example:

$$\begin{aligned} (\hat{E}_\mu \star \hat{E}_\nu)^{(1)} &= \hat{E}^{(1)}_\mu e_\nu + e_\mu \hat{E}^{(1)}_\nu + \frac{i}{2}\theta^{\alpha\beta}\partial_\alpha e_\mu \partial_\beta e_\nu \qquad (19) \\ &= -\frac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha, \partial_\beta(e_\mu e_\nu) + D_\beta(e_\mu e_\nu)\} + \frac{i}{2}\theta^{\alpha\beta}(D_\alpha e_\mu)(D_\beta e_\nu). \end{aligned}$$

Similarly one calculates all other products that appear in the action (15). Some examples:

$$
\begin{aligned}
(\hat{R}_{\mu\nu} \star \hat{E}_\rho \star \hat{E}_\sigma)^{(1)} &= \hat{R}^{(1)}_{\mu\nu}(e_\rho e_\sigma) + R_{\mu\nu}(\hat{E}_\rho \star \hat{E}_\sigma)^{(1)} + \frac{i}{2}\theta^{\alpha\beta}\partial_\alpha(R_{\mu\nu})\partial_\beta(e_\rho e_\sigma) \\
&= -\frac{1}{4}\theta^{\alpha\beta}\{\omega_\alpha, \partial_\beta(R_{\mu\nu}e_\rho e_\sigma) + D_\beta(R_{\mu\nu}e_\rho e_\sigma)\} \\
&\quad + \frac{i}{2}\theta^{\alpha\beta}(D_\alpha R_{\mu\nu})D_\beta(e_\rho e_\sigma) \\
&\quad + \frac{1}{2}\theta^{\alpha\beta}\{R_{\alpha\mu}, R_{\beta\nu}\}e_\rho e_\sigma + \frac{i}{2}\theta^{\alpha\beta}R_{\mu\nu}(D_\alpha e_\rho)(D_\beta e_\sigma), \\
(\hat{R}_{\alpha\beta} \star \hat{R}_{\mu\nu})^{(1)} &= -\frac{1}{4}\theta^{\kappa\lambda}\{\omega_\kappa, \partial_\lambda(R_{\alpha\beta}R_{\mu\nu}) + D_\lambda(R_{\alpha\beta}R_{\mu\nu})\} \\
&\quad + \frac{i}{2}\theta^{\kappa\lambda}(D_\kappa R_{\alpha\beta})(D_\lambda R_{\mu\nu}) + \frac{1}{2}\theta^{\kappa\lambda}(\{R_{\kappa\alpha}, R_{\lambda\beta}\}R_{\mu\nu} \\
&\quad + R_{\alpha\beta}\{R_{\kappa\mu, R_{\lambda\nu}}\}),
\end{aligned}
$$

The first order correction of the Einstein-Hilbert term is

$$
\begin{aligned}
S_{EH}^{(1)} &= \frac{1}{64\pi G_N}\epsilon^{\mu\nu\rho\sigma}\int \mathrm{d}^4x \mathrm{Tr}\left(\hat{R}_{\mu\nu}\star(\hat{E}_\rho\star\hat{E}_\sigma)\gamma_5\right)^{(1)} \\
&= -\frac{1}{256\pi G_N}\epsilon^{\mu\nu\rho\sigma}\theta^{\alpha\beta}\int \mathrm{d}^4x \mathrm{Tr}\,\gamma_5\Big(\{R_{\alpha\beta},R_{\mu\nu}\}e_\rho e_\sigma \\
&\quad -2\{R_{\alpha\mu},R_{\beta\nu}\}e_\rho e_\sigma - 2iR_{\mu\nu}(D_\alpha e_\rho)(D_\beta e_\sigma)\Big) \\
&\downarrow \\
S_{EH}^{(2)} &= -\frac{1}{512\pi G_N}\epsilon^{\mu\nu\rho\sigma}\theta^{\alpha\beta}\int \mathrm{d}^4x \mathrm{Tr}\,\gamma_5\Big(\{\hat{R}_{\alpha\beta}\stackrel{\star}{,}\hat{R}_{\mu\nu}\}\star\hat{E}_\rho\star\hat{E}_\sigma \\
&\quad -2\{\hat{R}_{\alpha\mu}\stackrel{\star}{,}\hat{R}_{\beta\nu}\}\star\hat{E}_\rho\star\hat{E}_\sigma - 2i\hat{R}_{\mu\nu}\star(D_\alpha\hat{E}_\rho)\star(D_\beta\hat{E}_\sigma)\Big)^{(1)}.
\end{aligned}
$$

Similarly to (17), one can also formulate recursive relations for the action . . . .

Inserting these expressions and calculating traces explicitly, we obtain

$$\begin{aligned}
S_{GB}^{(2)} &= -\frac{l^2}{1024\pi G_N}\theta^{\kappa\lambda}\theta^{\rho\sigma}\epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd}\int \mathrm{d}^4x\Big[R_{\alpha\beta}^{\ \ cd}R_{\mu\kappa}^{\ \ ab}R_{\nu\rho}^{\ \ mn}R_{\lambda\sigma mn} \\
&-\frac{1}{2}R_{\alpha\beta}^{\ \ cd}R_{\mu\kappa}^{\ \ ab}R_{\nu\lambda}^{\ \ mn}R_{\rho\sigma mn} + R_{\alpha\beta}^{\ \ mn}R_{\mu\kappa mn}R_{\nu\rho}^{\ \ ab}R_{\lambda\sigma}^{\ \ cd} \\
&+R_{\mu\rho}^{\ \ mn}R_{\nu\sigma mn}R_{\alpha\kappa}^{\ \ ab}R_{\beta\lambda}^{\ \ cd} - \frac{1}{2}R_{\alpha\kappa}^{\ \ ab}R_{\beta\lambda}^{\ \ cd}R_{\rho\sigma}^{\ \ mn}R_{\mu\nu mn}\Big],
\end{aligned} \tag{20}$$

$$\begin{aligned}
S_{CC}^{(2)} &= -\frac{1}{512\pi G_N l^2}\theta^{\kappa\lambda}\theta^{\alpha\beta}\int \mathrm{d}^4x e\Big(6R_{\kappa\alpha}^{\ \ ab}R_{\lambda\beta ab} - 3R_{\alpha\beta}^{\ \ ab}R_{\kappa\lambda ab} \\
&+4R_{\alpha\beta}^{\ \ \gamma\delta}(D_\kappa e_\mu)^a(D_\lambda e_\gamma)^b(e_a^\mu e_{\delta b} + e_b^\mu e_{\delta a}) \\
&-4R_{\alpha\beta}^{\ \ \gamma\delta}(D_\kappa e_\gamma)^a(D_\lambda e_\delta)_a + 4(D_\kappa D_\alpha e_\mu)^a(D_\lambda D_\beta e_\nu)^b(e_a^\mu e_b^\nu - e_b^\mu e_a^\nu) \\
&-8R_{\kappa\alpha}^{\ \ \gamma\delta}(D_\beta e_\gamma)^a(D_\lambda e_\delta)_a + 8R_{\kappa\alpha}^{\ \ \gamma\delta}(D_\beta e_\gamma)^a(D_\lambda e_\mu)^b(e_a^\mu e_{\delta b} + e_b^\mu e_{\delta a})\Big),
\end{aligned} \tag{21}$$

$$
\begin{aligned}
S_{EH}^{(2)} \quad = \quad & -\frac{1}{512\pi G_N}\theta^{\alpha\beta}\theta^{\kappa\lambda}\int d^4xe\Big(\frac{1}{2}R_{\kappa\lambda}{}^{\mu\nu}R_{\alpha\beta}{}^{\gamma\delta}R_{\mu\nu\gamma\delta} \\
& +R_{\kappa\lambda\rho\sigma}(\frac{1}{2}R_{\alpha\beta}{}^{\mu\nu}R_{\mu\nu}{}^{\rho\sigma}-2R_{\alpha\beta}{}^{\mu\sigma}R_{\mu}{}^{\rho}+\frac{1}{2}RR_{\alpha\beta}{}^{\rho\sigma} \\
& +4R_{\beta\nu}{}^{\rho\sigma}R_{\alpha}{}^{\nu}+4R_{\alpha}{}^{\rho}R_{\beta}{}^{\sigma}-4R_{\alpha\mu}{}^{\nu\rho}R_{\beta\nu}{}^{\mu\sigma}) \\
& -2R_{\kappa\lambda}{}^{\mu\nu}R_{\alpha\mu}{}^{\gamma\delta}R_{\beta\nu\gamma\delta}-RR_{\kappa\alpha}{}^{\gamma\delta}R_{\lambda\beta\gamma\delta} \\
& -R_{\mu\nu\rho\sigma}(2R_{\kappa\alpha}{}^{\mu\nu}R_{\lambda\beta}{}^{\rho\sigma}+4R_{\kappa\alpha}{}^{\mu\sigma}R_{\lambda\beta}{}^{\nu\rho}) \\
& -4R_{\alpha}{}^{\nu}R_{\kappa\beta}{}^{\gamma\delta}R_{\lambda\nu\gamma\delta}+2R_{\alpha\mu\rho\sigma}(2R_{\kappa\beta}{}^{\mu\nu}R_{\lambda\nu}{}^{\rho\sigma} \\
& -4R_{\lambda}{}^{\rho}R_{\kappa\beta}{}^{\mu\sigma}+4R_{\kappa\beta}{}^{\nu\rho}R_{\lambda\nu}{}^{\mu\sigma}+2R_{\kappa\beta}{}^{\rho\sigma}R_{\lambda}{}^{\mu}) \\
& +4e_b^{\mu}e_c^{\nu}((D_\kappa R_{\alpha\beta})^{mb}(D_\lambda R_{\mu\nu})^{c}{}_{m}-2(D_\kappa R_{\alpha\mu})^{mb}(D_\lambda R_{\beta\nu})^{c}{}_{m}) \\
& +(D_\kappa e_\rho)^m(D_\lambda e_\sigma)_m(2R_{\alpha\beta}{}^{\mu\nu}R_{\mu\nu}{}^{\rho\sigma}+8R_{\alpha\beta}{}^{\mu\rho}R_{\mu}{}^{\sigma}+2RR_{\alpha\beta}{}^{\rho\sigma} \\
& -8R_{\alpha\mu}{}^{\rho\sigma}R_{\beta}{}^{\mu}-8R_{\beta}{}^{\rho}R_{\alpha}{}^{\sigma}-8R_{\alpha\mu}{}^{\nu\rho}R_{\beta\nu}{}^{\mu\sigma}) \\
& -2(D_\kappa D_\alpha e_\rho)^c(D_\lambda D_\beta e_\sigma)^d(R(e_c^{\rho}e_d^{\sigma}-e_c^{\sigma}e_d^{\rho})-3R_{\nu}{}^{\rho}e_c^{\nu}e_d^{\sigma} \\
& +3R_{\nu}{}^{\sigma}e_c^{\nu}e_d^{\rho}+R_{\nu}{}^{\rho}e_c^{\sigma}e_d^{\nu}-R_{\nu}{}^{\sigma}e_c^{\rho}e_d^{\nu}+2R_{\mu\nu}{}^{\rho\sigma}e_c^{\mu}e_d^{\nu}) \\
& +4(D_\lambda e_\sigma)^a R_{\mu\nu}{}^{bm}(D_\alpha e_\rho)_m(R_{\kappa\beta}{}^{\mu\nu}(e_a^{\rho}e_b^{\sigma}-e_a^{\sigma}e_b^{\rho}) \qquad (22) \\
& +2R_{\kappa\beta}{}^{\mu\sigma}(e_a^{\nu}e_b^{\rho}-e_a^{\rho}e_b^{\nu})-2R_{\kappa\beta}{}^{\mu\rho}(e_a^{\nu}e_b^{\sigma}-e_a^{\sigma}e_b^{\nu})+2R_{\kappa\beta}{}^{\rho\sigma}e_a^{\mu}e_b^{\nu})\Big).
\end{aligned}
$$

Conclusions & Outlook

- NC gravity action
  -2nd order calculated explicitly; written in a manifestly gauge covariant way; correction terms are functions of $R_{\mu\nu}^{\ \ ab}$ and $e_\mu^{\ a}$, their contractions and their covariant derivatives
  -EOM? Can one go from the 1st order formalism to the 2nd order formalism? Does NC generates torsion?
  -phenomenological consequences, connection with $f(R)$ theories?
  -understanding of the model: corrections to GR solutions (black holes, gravitational waves,...)
- future investigation
  -NC $SO(2,3)_\star$ symmetry and SSB in NC theory
  -SW map for composite fields and renormalization of gauge theories