Kerr-Newman gravity beyond quantum theory: Electron as a closed heterotic string

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Based on:

A.B., *Gravity beyond Quantum theory: Gravitational strings..* [arXiv:1109.3872];
A.B. *Calabi-Yau twofold from the Kerr theorem*,[arXiv:1203.4210].

Conflict between Quantum theory and Gravity goes along many lines.
One of them is the quantum statement on the *pointlike electron*.

Absence of the space-time structure of electron contradicts to gravity.

On the other hand,
the consistent with gravity electron's background is given by the Kerr-Newman solution, which displays an *extended structure of the Compton size,* which is much bigger than the Planck scale!

Resolution of this conflict can give a clue to Quantum Gravity!

KERR-NEWMAN SPINNING PARTICLE.

The idea that black holes are related with elementary particles and string theory is not new, [G.'t Hooft (1990), A. Sen (1995), C.F.E. Holzhey and F. Wilczek (1992), A.Salam and J. Strathdee (1976)].

The Kerr-Newman (KN) solution has gyromagnetic ratio $g = 2$ as that of the Dirac electron (Carter 1968). Because of that the experimentally observable parameters of the electron (mass m, spin J, charge e and magnetic moment μ) determine *unambiguously* that its background should be the KN solution!

Spin of the electron is very high, $a = J/m >> m$, and the black hole horizons disappear. The source of the KN spinning particle appears as a

NAKED SINGULAR RING OF THE COMPTON RADIUS $a = \hbar/2m$.

– the spacetime has a topological peculiarity at the Compton distance $r_c = a = \frac{\hbar}{2m}$, which may be interpreted as *a closed string,* [AB & Ivanenko (1975)].

There appear questions:

Why Quantum theory does not feel this topological defect?

Why the singular ring at the Compton distance is unobservable?

Second stringy structure – a COMPLEX STRING appears in the *complex Kerr geometry. Striking similarities with some structures of the superstring theory.* In particular, orientifolding the complex string generates K3-surface on projective twistor space.

Complexification as alternative to compactification.

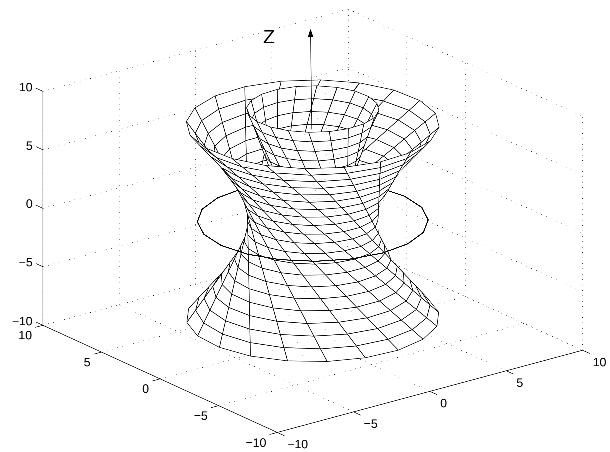
REAL structure of the Kerr-Newman solution: Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}, \tag{1}$$

and electromagnetic (EM) vector potential is

$$A^\mu_{KN} = Re \frac{e}{r + ia\cos\theta} k^\mu. \tag{2}$$

Gravitational and EM fields are concentrated near the Kerr singular ring.



The Kerr ring forms a branch line of space. The KN geometry is TWOSHEETED! Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC),

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv), \quad Y(x) = e^{i\phi} \tan\frac{\theta}{2}, \tag{3}$$

where $Y(x)$ is projective angular coordinate, and

$$\zeta = (x+iy)/\sqrt{2},\ \bar\zeta = (x-iy)/\sqrt{2},\ u = (z-t)/\sqrt{2},\ v = (z+t)/\sqrt{2}$$

are the null Cartesian coordinates.

Kerr congruence is controlled by the

KERR THEOREM:

The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0 \ , \qquad (4)$$

where F is an arbitrary analytic function of the

projective twistor coordinates

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar\zeta\}. \qquad (5)$$

The Kerr theorem is a practical tool for obtaining exact solutions:

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x)$$

For the Kerr-Newman solution function F is quadratic in Y, which yields TWO roots $Y^\pm(x) \Rightarrow$ two congruences!

KN solution brings in the NEW DIMENSIONAL PARAMETER $a = J/m$, which is the Compton length for $J \sim \hbar$!

Kerr's ring is a branch line of space on two sheets: "negative (–)" and "positive (+)" where the fields change their directions. In particular,

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \qquad g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}. \qquad (6)$$

Twosheeted mystery creates the problem of the source of the KN solution.

Kerr's oblate spheroidal coordinates $x + iy = (r + ia)e^{i\phi} \sin\theta, \quad z = r\cos\theta$, cover spacetime twice: disk $r = 0$ separates the 'out'-sheet $r > 0$, from the 'in'-sheet $r < 0$.

(a) *Closed 'Alice' string*: AB 1974, Gravitational strings: D.Ivanenko & AB 1975, W.Israel 1977, Heterotic string of the low energy string theory, A. Sen (1992), AB(1993-2011).

(b) *Relativistically rotating disk.* H.Keres (1967), W.Israel (1969), Hamity, I.Tiomno (1973).

H. Keres, [ZhETP, v.52, iss.3, 1967, pp.768-779].

ON PHYSICAL INTERPRETATION OF SOLUTIONS OF THE EINSTEIN EQUATIONS

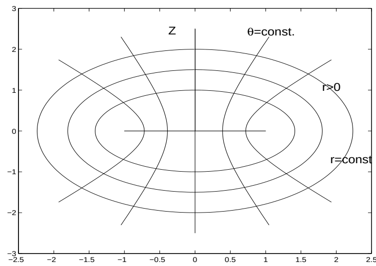
... the Kerr solution is considered, and it is shown that the corresponding gravitational field is by a disk-like layer of the negative mass, on the boundary of which lies a (closed) *line of positive mass with an infinite linear density...*

...the real potential

$$\Phi = Re(m/\sigma), \tag{7}$$

where $\sigma^2 = x^2 + y^2 + (z + ia)^2, \quad \sigma = r + ia\cos\theta,$

r and θ are the Kerr oblate spheroidal coordinates.



"...the function σ^{-1} and its first derivatives are continuous over all space, beside the disk $z = 0,\ x^2 + y^2 \leq a^2$ (or $r = 0, \theta \leq \pi/2$). By crossing the disk the derivatives have a jump discontinuity. Consequently, the sources of gravitational field lie on the disk...".

The H. Keres was first who considered the Kerr disk and anticipated the appearance of the closed string on the border of the Kerr disk. Carter, and then Israel interpreted it as a model of electron. Further developments of the problem of Kerr's source:

V. Hamity (1976) *Phys. Lett.* A **56** p.77 – *the Kerr disk is relativistically rotating!*

- **(c)** ***Relativistically rotating membrane,*** C.López (1983) .
- **(d)** ***Gravitating soliton: vacuum bubble bounded by membrane***, AB (2010).
- **(e)** ***Complex KN source as a COMPLEX STRING***, AB (1993-2012).

Complex Shift. Appel solution of 1887! A point-like charge e, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, -ia)$, gives the potential (9).

Pointlike ' IMAGE' of the electron in Quantum theory may be created by the ***Lorentz contraction of the relativistically rotating string*** *at the boundary of M2-brane.*

FUNDAMENTAL STRINGS as soliton-like classical solutions in the effective string field theory. [Dabholkar at.al, NPB 1990]. Classical singular solutions as fields around a HETEROTIC STRING [E. Witten, Phys.Lett.B 1985]. MACROSCOPIC CHARGED HETEROTIC STRING, [A. Sen, NPB 1992-1993]: bosonic zero modes of the 4D solutions correspond to bosonic degrees of freedom of heterotic string. PP-wave solutions – ***critical heterotic string theory*** in 4D with the extra six dimensions compactified.

Solutions to Einstein's eqs. are solutions to low energy string theory:

$$S = \int d^4x\sqrt{-g}(R - 2(\partial\phi)^2 - e^{-2\phi}F^2 - \frac{1}{2}e^{4\phi}(\partial a)^2 - aF\tilde{F})$$

Extra fields: axion a and dilaton ϕ. The Kerr solution corresponds to: $a = F^2 = \phi = 0 \Rightarrow$ ***Kerr's ring is a string!***

PP-WAVES as exact solutions! [Horowitz&Steif, 1990, A.Tseytlin, 1993].

Traveling waves as modes of string excitations, D.Garfinkel (PRD 1992).

Kerr-Sen solution to low energy string theory: The Kerr solution with the nontrivial axion and dilaton fields, A. Sen (PRL 1992).

The Kerr SINGULAR RING is a 'closed' heterotic string. The field around Kerr-Sen solution to low energy string theory is similar to the Sen solution for HETEROTIC STRING. [AB PRD 1995]. Only left fermion modes – lightlike circular currents.

REGULAR BUBBLE – ELECTRON AS A GRAVITATING SOLITON.

SOLITON is formed by the disklike vacuum bubble surrounding the Kerr ring [AB, 2010]. Interior of bubble is flat, $g_{\mu\nu} = \eta_{\mu\nu}$.

Especial domain wall model interpolates between the external KN 'vacuum state', $\mathbf{V_{ext}} = \mathbf{0}$, and the internal false vacuum state, $\mathbf{V_{int}} = \mathbf{0}$. Boundary of the domain wall, $H = 0 \ \Rightarrow r = e^2/2m$, determines the vector potential cut-off:

$$r_{min} = e^2/2m \Rightarrow A_{max} = \frac{2m}{e}. \tag{8}$$

Regularization is performed by the Higgs mechanism of broken symmetry. The HIGGS vacuum condensate inside the bubble $\Phi = |\Phi| \exp\{i\chi\}$ interacts with the KN electromagnetic field A_μ, in agreement with the equations $\Box A_\mu = I_\mu = e|\Phi|^2(\chi,_\mu + eA_\mu)$, and regularizes A_μ, pushing it out to the bubble boundary.

Inside the bubble, $|\Phi| > 0, \quad I_\mu = 0$, and $\Box A_\mu = 0, \quad \chi,_\mu + eA_\mu = 0$ – gradient of the phase of the Higgs field $\chi,_\mu$ "eats up" the KN electromagnetic(EM) field A_μ, expelling the field strength and currents to the string-like boundary of the bubble.

(i) the Kerr ring is regularized, forming a closed relativistic string of the Compton radius on the border of the disklike bubble,

(ii) the KN electromagnetic potential forms on the perimeter of the bubble the quantized Aharonov-Bohm-Wilson loop $\oint eA_\varphi d\varphi = -4\pi ma$, which results in ***quantization of the total spin,*** $J = ma = n\hbar/2, \ n = 1, 2, 3, ...,$

(iii) the Higgs condensate is to be oscillating with the frequency $\omega = 2m$.

Complex Structure of the Kerr geometry.

Complex shift of the Coulomb solution. Appel solution 1887!

A point-like charge e, shifted along the complex z-axis $(x_0, y_0, z_0) = (0, 0, -ia)$, gives a real potential

$$\Phi = Re(m/\sigma), \tag{9}$$

where $\sigma^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = x^2 + y^2 + (z + ia)^2$,
or $\sigma = r + ia \cos\theta$, where r and θ are Kerr's oblate spheroidal coordinates.
Parallel with Harald Keres representation [ZhETP v.52, 1967, p.768]!

There is an exact correspondence between Appel's complex shift and Kerr's geometry. The Kerr-Newman solution is generated by a complex source!
Newman's retarded-time construction (1973).
New objects: ***Complex light cones*** with the vertexes on the *complex world-line* $x_0^\mu \in CM^4$:

$(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$, splits into two families of the "left" and "right" complex null planes: $x_L^\mu = x_0^\mu(\tau) + \alpha e^{1\mu} + \beta e^{3\mu}$ e^1 and e^3, and $x_R^\mu = x_0^\mu(\tau) + \alpha e^{2\mu} + \beta e^{3\mu}$, spanned by null tetrad e^a, $(e^a)^2 = 0$. Twistor coordinates!

The Kerr congruence arises from real slices of the family of the "left" null planes ($Y = const.$) of the complex light cones whose vertices lie at a complex world-line $x_0(\tau)$.

Complex string as source of the Kerr geometry. AB [gr-qc/9303003, 1203.4210]. Kerr's source can be considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ parametrized by complex time $\tau = t + i\sigma$.
There appears a the 'Left' and 'Right' retarded times as intersections of the 'Left' and 'Right' complex null planes with the 'Left' and 'Right' complex world-lines.

The complex world-line parametrized by $\tau = t + i\sigma$ forms a world-sheet. [Earlier discussion of the complex world-line as a string also by Oogury and Vafa (1991).]

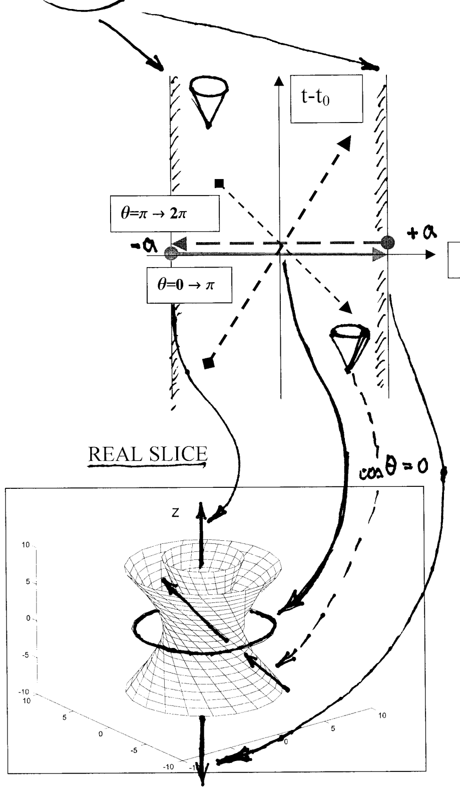
Open Euclidean string $X_L(\tau_L) \equiv X_L^\mu(t_L + i\sigma_L)$ with the ends at $\sigma = \pm a$.

Real Kerr's geometry appears as real slice of this complex structure.

Complex World line as a Complex String $X_\mu(\tau)$

Plane of the Complex time $\tau = t + i\sigma = t + i\,a\cos\theta$

End points of the open string $\sigma = -a,\ +a \Leftrightarrow \theta = -\pi,\ +\pi$



Along with the considered complex world-line (say 'Left'), there is a complex conjugate world-line, $X_L(\tau_L)$ and $X_R(\tau_R)$.

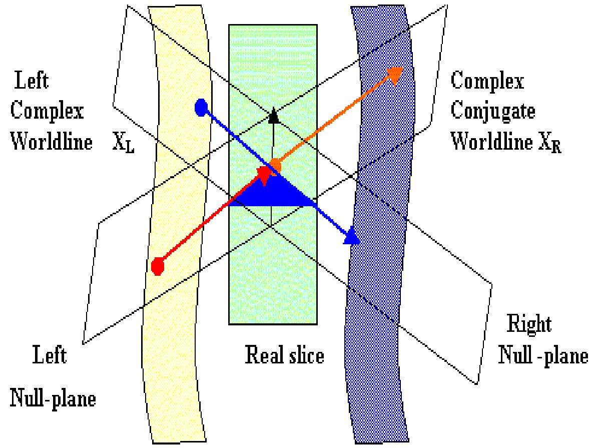


Figure 1: **Complex light cone at a real point x. The adjoined to congruence Left and Right complex null planes. Four roots: X_L^{adv}, X_L^{ret} and X_R^{adv}, X_R^{ret} which are related by crossing symmetry.**

Kerr theorem $\Rightarrow$ twoseetedness of Kerr geometry. Function $F(T^A)$ is quadratic in Y. It is a *quadric* in twistorial CP^3,

$$F = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu), \tag{10}$$

and has a non-degenerate determinant $\Delta = (B^2 - 4AC)^{1/2}$ which determines the complex radial distance

$$\tilde{r} = -\Delta = -(B^2 - 4AC)^{1/2}. \tag{11}$$

This case is easily resolved and yields two solutions

$$Y^{\pm}(\mathrm{x}) = (-\mathrm{B} \mp \tilde{\mathrm{r}})/2\mathrm{A}, \tag{12}$$

The corresponding two congruences are analytically related by the transfer from the positive ($r > 0$) to the negative ($r < 0$) sheet.

Antipodal map: $Y^+ \to -1/\bar{Y}^-$.

Left and Right complex structures form an

wordsheet orientifold of the complex string.

$\Omega = Compl.\ Conj. + Revers\ of\ radial\ coordinate.$

Orientifold projection of the Kerr background matches the Kerr congruences Y^+ and Y^-.

There appears matching of the retarded and advanced fields.

$\Omega + Antipodal\ map.$

Kerr theorem for multi-particle KS space-times.

Selecting an isolated i-th particle with parameters q_i, one can obtain the roots $Y_i^{\pm}(x)$ of the equation $F_i(Y|q_i) = 0$ and express F_i in the form

$$F_i(Y) = A_i(x)(Y - Y_i^+)(Y - Y_i^-). \tag{13}$$

Then, the $(+)$ or $(-)$ root $Y_i^{\pm}(x)$ determines congruence $k_\mu^{(i)}(x)$ and consequently, the Kerr-Schild metric

$$g_{\mu\nu}^{(i)} = \eta_{\mu\nu} + 2h^{(i)} k_\mu^{(i)} k_\nu^{(i)}, \tag{14}$$

and finally, the function $h^{(i)}(x)$ may be expressed in terms of $\tilde{r}_i = -d_Y F_i$, as follows

$$h^{(i)} = \frac{m}{2}\left(\frac{1}{\tilde{r}_i} + \frac{1}{\tilde{r}_i^*}\right) + \frac{e^2}{2|\tilde{r}_i|^2}. \tag{15}$$

Electromagnetic field is given by the vector potential

$$A_\mu^{(i)} = \Re e(e/\tilde{r}_i) k_\mu^{(i)}. \tag{16}$$

For a system of k particles we form the function F as a product of the known blocks $F_i(Y)$,

$$F(Y) \equiv \prod_{i=1}^{k} F_i(Y). \tag{17}$$

The solution of the equation $F = 0$ acquires $2k$ roots $Y_i^{\pm}$, and the twistorial space turns out to be multi-sheeted.

The twistorial structure on the i-th $(+)$ or $(-)$ sheet is determined by the equation $F_i = 0$ and does not depend on the other functions $F_j, \quad j \neq i$. Therefore, the particle i does not feel the twistorial structures of other particles. Similar, the condition for singular lines $F = 0, \ d_Y F = 0$ acquires the form

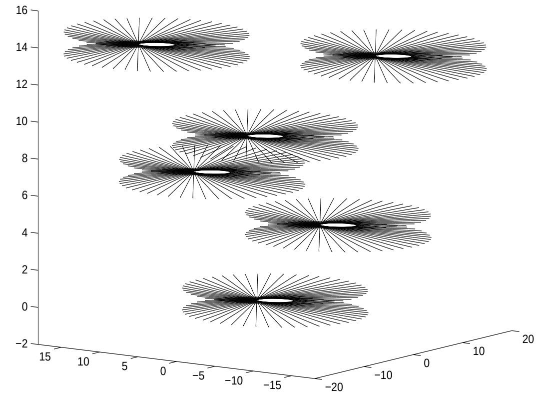


Figure 2: Multi-sheeted twistor space over the auxiliary Minkowski space-time of the multi-particle Kerr-Schild solution. Each particle has twofold structure.

$$\prod_{l=1}^{k} F_l = 0, \qquad \sum_{i=1}^{k} \prod_{l\neq i}^{k} F_l d_Y F_i = 0 \tag{18}$$

and splits into k independent relations

$$F_i = 0, \quad \prod_{l\neq i}^{k} F_l d_Y F_i = 0. \tag{19}$$

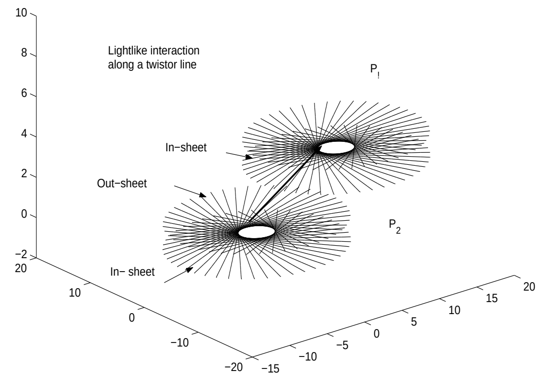


Figure 3: Schematic representation of the lightlike interaction via a common twistor line connecting out-sheet of one particle to in-sheet of another.

The number of surrounding particles and number of blocks in the generating function F may be assumed countable. In this case the multi-sheeted twistorial space-time will possess the properties of the multi-particle Fock space.

The Left and Right structures by excitations should be considered as independent and generated by different KN sources, which corresponds to two-particle KN system with *quadratic* generating functions of the Kerr theorem $F_1(T)$ and $F_2(T)$, determined on the projective twistor space CP^3.

Kerr Theorem $\Rightarrow$ the joint twistor system is to be described by the generating function $F_{12}(T) = F_1(T) \cdot F_2(T)$. The corresponding equation

$$F_{12}(T) = F_1(T) \cdot F_2(T) = 0,$$

is $QUARTIC$ on the projective twistor space, and therefore the complex string forms a *Calabi-Yau twofold embedded in the projective twistor space* [arXiv:1203.4210].

Product of the KN closed heterotic string on the KN complex string creates the M2-brane – which corresponds to the relativistically rotating BUBBLE source of KN spinning particle.

Striking parallelism with the superstring theory.
In the same time there are very essential differences:

- the space-time is four-dimensional – a "compactification without compactification",
- a natural consistency with gravity,
- characteristic parameter of the Kerr strings $a = \hbar/m$ corresponds to Compton scale, which is closer to particle physics vs. the Planck scale of superstring theory.

SHAPE OF THE REGULARIZED KN SOLUTION is
RELATIVISTICALLY ROTATING M2-brane OF COMPTON SIZE!
THE POINT-LIKE IMAGE OF ELECTRON may be caused by the LORENTZ CONTRACTION of the relativistically rotating closed string!!!

THANK YOU FOR YOUR ATTENTION!