Gauge invariant perturbations in non-local gravity models*

Alexey S. Koshelev†
Theoretische Natuurkunde, Vrije Universiteit Brussel and
The International Solvay Institutes,
Pleinlaan 2, B-1050 Brussels, BELGIUM

Abstract

We systematically derive the closed system of equations for linear scalar perturbations in stringy motivated non-local cosmological models. All the equations are written in manifestly gauge invariant variables.

1. Introduction and summary

Recent observations [1] strongly support that primordial inflation is the theoretical explanation of how the currently observed Universe was formed at the early stages. Alongside with observations theoretical approaches also show how nice inflation can be connected to the nucleosynthesis and subsequent appearance of the particle standard model. A number of inflationary scenarios are reviewed in [2] and references therein, for instance.

Even though inflation is a great model for many reasons it has problems one of which is the lack of the UV completion. To be more precise it is not UV-complete in the framework of the Einstein's General Relativity (GR) since geodesics are not past-complete. This is a general statement and it is known as the Big Bang singularity elaborated in [3, 4, 5]. One can find that alternatives to Big Bang such as "emergent" Universe or bouncing Universe [6] hit the singularity theorem by Hawking and Penrose [7] as long as we are in GR and the space-time is of the Friedmann–Lemaître–Robertson–Walker (FLRW) type.

One of the possible resolution is a modification of GR. This can be done in general in a number of ways and one may have an insight in this using the review paper [8] and references therein, for example. It is inevitable

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† e-mail address: alexey.koshelev@vub.ac.be

that any modification of gravity introduces higher derivatives and only special structures like Gauss-Bonnet term or Lovelock terms in general [8, 9] preserve the second order of the equations of motion but this is applicable only in more than 4 dimensions. On the other hand finite higher derivatives lead to ghosts due to the Ostrogradski theorem [10]. Having all orders of higher derivatives may open a way to evade the Ostrogradski statement and a successful attempt in this direction was made considering a special class of gravity modifications where higher curvature corrections are accompanied with non-local operators [11, 12]. Analysis in those papers shows how one can construct a ghost-free and asymptotically free modification of GR featuring a non-singular bouncing solution, which resembles GR in the infra-red limit. Note that similar approaches involving non-local models were used in other cosmological and gravity contexts in the literature [13, 14, 15, 16, 17]. The further study of this model [18] has shown that the model features the expected perturbative spectrum at late times and is stable with respect to small isotropic inhomogeneous perturbations during the bounce phase and in parallel a more general model [19] (see also [20]) was proposed and considered in the Minkowski background.

Absolutely the non-local operators is what makes these models novel and the operators in question are of type of analytic functions of the covariant d'Alembertian operator $\Box$, i.e. $\mathcal{F}(\Box)$. Note that other theoretically motivated operators such as $1/\Box$ were considered, for instance, in [21, 22] and references therein. It is important that the initial inspiration for introducing analytic $\mathcal{F}(\Box)$ operators comes from string field theory (SFT) models because SFT as the whole theory is a UV-complete non-perturbative description of strings. We refer the reader to more stringy oriented literature [28, 23, 24, 25, 26, 27] for the more comprehensive overview of this aspect. A decent progress was achieved in studying non-local scalar field models derived from SFT in the cosmological context [29]-[32] as well as exploration of other aspects of this type of models including their thermodynamics [33, 34, 35]. The major question of rigorous derivation of the modified GR action involving the non-local operators of interest from the scratch, i.e. from the closed SFT, is still awaiting for its resolution and this is beyond the scope of our present study.

In [18] a lot of technical issues were solved for a model which contains the scalar curvature squared non-local term. The main focus there is perturbations around a bouncing solution. In [19] non-local terms containing the Ricci and Riemann tensors squared were added but only analyzed around the Minkowski background. In [36] the efforts of [18, 19] works were joined and the analysis was extended for a much more general model.

In the present paper we derive the closed system of perturbation equations for the most general non-local quadratic in curvature gravitational action. We confine ourselves by considering the FLRW type of the metric and positive cosmological term Λ. Bouncing or cyclic models of the Universe [37] try to establish a connection between the perturbative spectrum during or prior the bounce phase and the spectrum observed in CMB. In order to do the same in stringy motivated non-local models an appropriate technique must be developed. This provides us with the additional strong motivation for the present study.

As the final result we find out a system of equations which can be solved at least numerically. This is a closed system of equations obtained for the first time for this class of models. Surely, we expect a number of applications of this result since it opens possibilities for the deeper study of the perturbation spectrum in such non-local cosmologies.

2. Action and equations of motion

We focus on the model described by the following non-local action

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{\lambda}{2}\mathcal{R}_2 - \Lambda\right), \tag{1}$$

where

$$\mathcal{R}_2 = R\mathcal{F}_1(\Box)R + R^\mu_\nu\mathcal{F}_2(\Box)R^\nu_\mu + C_{\mu\nu\alpha\beta}\mathcal{F}_4(\Box)C^{\mu\nu\alpha\beta},$$

and hence we limit ourselves with $O(R^2)$ corrections. Here the dimensionality is manifest and in the sequel all the formulae are written having 4 dimensions in mind, M_P is the Planckian mass, Λ is a cosmological constant and λ is a dimensionless parameter measuring the effect of the $O(R^2)$ corrections. The most novel and crucial for our analysis ingredients are the functions of the covariant d'Alembertian operator $\mathcal{F}_I$. For simplicity to avoid extra complications we assume that these function are analytic with real coefficients f_{I_n} in Taylor series expansion $\mathcal{F}_I = \sum_{n\geq 0} f_{I_n}\Box^n/M_*^{2n}$. The new mass scale determines the characteristic scale of the gravity modification. We assume it universal for all $\mathcal{F}_I$ and refer the reader to [11] for a detailed discussion of this new physics parameter. Also apart from the canonical usage of the Riemann tensor we use the Weyl tensor $C^\mu_{\alpha\nu\beta}$ which is coming from the Ricci decomposition

$$C^{\mu\alpha}{}_{\nu\beta} = R^{\mu\alpha}{}_{\nu\beta} - \frac{1}{2}(\delta^\mu_\nu R^\alpha_\beta - \delta^\mu_\beta R^\alpha_\nu + R^\mu_\nu\delta^\alpha_\beta - R^\mu_\beta\delta^\alpha_\nu) + \frac{R}{6}(\delta^\mu_\nu\delta^\alpha_\beta - \delta^\mu_\beta\delta^\alpha_\nu).$$

In this formula we use slightly unusual position of indexes which is useful in performing further computations. The reason to use the Weyl tensor is because $C^\mu_{\alpha\nu\beta} = 0$ on a conformally flat manifold which is the case for the FLRW metric. We are focused on the FLRW cosmologies and thus will benefit out of this. Indeed, it means that the Weyl tensor squared does not show up in the background at all and only becomes relevant in perturbations. Moreover, even in perturbations the only non-vanishing contribution is the one where both Weyl tensors are perturbed and the non-local functions $\mathcal{F}_4$ takes its background form.

This action appears in [19] (eq. (27)) and is the most general covariant non-local action up to the square in curvature terms and analytic operator functions $\mathcal{F}_I$. We have however less terms because possible contractions of the covariant derivatives with the Ricci tensor can be eliminated thanks to the Bianchi identity or converted to the higher order in curvature terms using the commutation relations for the covariant derivatives. Also questions of a ghost-free gravity modification are addressed in [38] using different action still using non-local operators constructed out of the d'Alembertian operator. Furthermore we note that working in 4 dimensions we can assume $f_{20} = 0$ because using that the Gauss-Bonnet scalar is a total derivative and combining this with the Ricci decomposition we can have only R^2 when no d'Alembertian operators are in between. In other words among the terms without d'Alembertian operator insertions only R^2 survives on the FLRW backgrounds. This does not work for non-constant terms in $\mathcal{F}_I$ though.

2.1. Model reformulation using $\tilde{G}^\mu_\nu$

The first useful technical step is a passage from the Ricci tensor to the traceless analog of the Einstein tensor $\tilde{G}^\mu_\nu$. We mention in this regard that appearance of a combination $\mathcal{F}_1 + \frac{1}{4}\mathcal{F}_2$ is not spontaneous because we can rewrite the initial action (1) in terms of $\tilde{G}^\mu_\nu$ as follows

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{\lambda}{2}\tilde{\mathcal{R}}_2 - \Lambda\right), \tag{2}$$

where

$$\tilde{\mathcal{R}}_2 = R\tilde{\mathcal{F}}_1(\Box)R + \tilde{G}^\mu_\nu\mathcal{F}_2(\Box)\tilde{G}^\nu_\mu + C_{\mu\nu\alpha\beta}\mathcal{F}_4(\Box)C^{\mu\nu\alpha\beta},$$

and we have used that $\tilde{G}^\mu_\nu$ is traceless and have defined $\mathcal{F}_1(\Box) + \frac{1}{4}\mathcal{F}_2(\Box) = \tilde{\mathcal{F}}_1(\Box)$. Equations of motion for action (2) can be derived by the straightforward computation

$$\begin{aligned}
M_P^2 G^\mu_\nu = {} & T^\mu_\nu - \Lambda\delta^\mu_\nu - \\
& - 2\lambda\tilde{G}^\mu_\nu\tilde{\mathcal{F}}_1(\Box)R + 2\lambda(\nabla^\mu\partial_\nu - \delta^\mu_\nu\Box)\tilde{\mathcal{F}}_1(\Box)R - \frac{1}{2}\lambda R\mathcal{F}_2(\Box)\tilde{G}^\mu_\nu - \\
& - 2\lambda\tilde{G}^\mu_\beta\mathcal{F}_2(\Box)\tilde{G}^\beta_\nu + \frac{\lambda}{2}\delta^\mu_\nu\tilde{G}^\alpha_\beta\mathcal{F}_2(\Box)\tilde{G}^\beta_\alpha + \\
& + 2\lambda\left(\nabla_\rho\nabla_\nu\mathcal{F}_2(\Box)\tilde{G}^{\mu\rho} - \frac{1}{2}\Box\mathcal{F}_2(\Box)\tilde{G}^\mu_\nu - \frac{1}{2}\delta^\mu_\nu\nabla_\sigma\nabla_\rho\mathcal{F}_2(\Box)\tilde{G}^{\sigma\rho}\right) + \\
& + \lambda\mathcal{L}_1{}^\mu_\nu - \frac{\lambda}{2}\delta^\mu_\nu\big(\mathcal{L}_1{}^\sigma_\sigma + \bar{\mathcal{L}}_1\big) + \lambda\mathcal{L}_2{}^\mu_\nu - \frac{\lambda}{2}\delta^\mu_\nu\big(\mathcal{L}_2{}^\sigma_\sigma + \bar{\mathcal{L}}_2\big) + 2\lambda\tilde{\Delta}^\mu_\nu + 2\lambda\mathcal{C}^\mu_\nu,
\end{aligned} \tag{3}$$

where we have defined:

$$\mathcal{L}_1{}^\mu_\nu = \sum_{n=1}^\infty \tilde{f}_{1n}\sum_{l=0}^{n-1}\partial^\mu R^{(l)}\partial_\nu R^{(n-l-1)},\ \ \bar{\mathcal{L}}_1 = \sum_{n=1}^\infty \tilde{f}_{1n}\sum_{l=0}^{n-1} R^{(l)}R^{(n-l)},$$

$$\mathcal{L}_2{}^\mu_\nu = \sum_{n=1}^\infty f_{2n}\sum_{l=0}^{n-1}\nabla^\mu\tilde{G}^{(l)}{}^\alpha_\beta\nabla_\nu\tilde{G}^{(n-l-1)}{}^\beta_\alpha,\ \ \bar{\mathcal{L}}_2 = \sum_{n=1}^\infty f_{2n}\sum_{l=0}^{n-1}\tilde{G}^{(l)}{}^\alpha_\beta\tilde{G}^{(n-l)}{}^\beta_\alpha,$$

$$\tilde{\Delta}^{\mu}_{\nu} = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} \nabla_{\beta}[\tilde{G}^{(l)\beta}_{\ \ \gamma} \nabla^{\mu} \tilde{G}^{(n-l-1)\gamma}_{\ \ \nu} - \nabla^{\mu} \tilde{G}^{(l)\beta}_{\ \ \gamma} \tilde{G}^{(n-l-1)\gamma}_{\ \ \nu}],$$

and $\tilde{f}_{1n}$ are coefficients of the Taylor expansion of function $\tilde{\mathcal{F}}_1$. The Weyl tensor related part may have an impact now since we are going to consider perturbations. One can find the relevant part of it is

$$\mathcal{C}^{\mu}_{\nu} = (R_{\alpha\beta} + 2\nabla_{\alpha}\nabla_{\beta})\,\mathcal{F}_4(\Box) C_{\nu}^{\ \alpha\beta\mu}.$$

Saying relevant we mean only the piece which is obtained by the variation of one of the Weyl tensor factors in the action. Then we are left with only one Weyl tensor as it is obvious from the latter formula and further perturbation of this remaining Weyl tensor factor may produce a non-zero contribution to the perturbation equations.

The trace equation becomes

$$\begin{aligned} - M_P^2 R = T - 4\Lambda - 6\lambda\Box\tilde{\mathcal{F}}_1(\Box)R - \lambda(\mathcal{L}_1 + 2\bar{\mathcal{L}}_1) - \\ - 2\lambda\nabla_{\rho}\nabla_{\mu}\mathcal{F}_2(\Box)\tilde{G}^{\mu\rho} - \lambda(\mathcal{L}_2 + 2\bar{\mathcal{L}}_2) + 2\lambda\tilde{\Delta}, \end{aligned} \tag{4}$$

and the Weyl tensor related term $\mathcal{C}$ turns out to be zero thanks to the full tracelessness of the Weyl tensor.

This form of action and equations of motion also turns out to be beneficial for studying perturbations.

3. GR Background

Here we systematically give the important notation to the background level for studying perturbations.

We work in 4 dimensions, indexes μ, ν run from 0 to 3, G is the Newton constant and the signature is $(-,+,+,+)$. Indexes a, b will be used for the spatial coordinates. The metric is chosen to give FLRW background

$$ds^2 = a(\eta)^2(-d\eta^2 + g^{(3)}_{ab} dx^a dx^b), \tag{5}$$

where η is the conformal time related to the cosmic one as $ad\eta = dt$ and a is the scale factor. In the sequel we will work in the spatially flat universe so that $g^{(3)}_{ab} = \delta_{ab}$. This corresponds to setting $K = 0$ where K is spatial curvature.

We start with Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{6}$$

accompanied by the conservation equation

$$D_{\mu} T^{\mu}_{\nu} = 0. \tag{7}$$

where D_μ is a covariant derivative.

The total background stress tensor is obviously diagonal of the form of a perfect fluid parameterized as follows

$$T^0_0 = -\rho, \qquad T^a_b = p\delta^a_b, \tag{8}$$

where ρ is the energy density and p is the pressure density. Background equations of motion are easily obtained to be

$$3H^2 = 8\pi G\rho, \qquad \dot{H} = -4\pi G(\rho + p), \qquad \dot{\rho} + 3H(\rho + p) = 0. \tag{9}$$

Here dot denotes a derivative w.r.t. the cosmic time t and $H \equiv \dot{a}/a$. The following notations will be used in the sequel:

$$w \equiv p/\rho, \qquad c^2 \equiv \dot{p}/\dot{\rho}.$$

w is the equation of state parameter, c^2 is the speed of sound. Constant w obviously results in $c^2 = w$. We name it an ideal perfect fluid. One useful relation is

$$\dot{w} = -3H(1 + w)(c^2 - w).$$

We assume that in general n non-interacting fluids present resulting in individual conservation equations

$$D_\mu T_{(i)}{}^\mu_\nu = 0. \tag{10}$$

Since there is no interaction among fluids except the minimal coupling we can introduce a convenient new index I which runs all values of i plus zero corresponding to collective quantities so that $\psi_{(0)} \equiv \psi$ for some variable ψ. Using it we can write equations (7) and (10) in a unified form

$$D_\mu T_{(I)}{}^\mu_\nu = 0. \tag{11}$$

Although in this particular case $T^\mu_\nu \equiv T_{(0)}{}^\mu_\nu = \sum_i T_{(i)}{}^\mu_\nu$ in general we shall use for some variables notations such that collective quantities are not just sums of individual ones. ρ and p are also easily decomposed into individual ones as $\rho \equiv \rho_{(0)} = \sum_i \rho_{(i)}, \quad p \equiv p_{(0)} = \sum_i p_{(i)}$. The following additional notations will be used

$$w_{(I)} \equiv p_{(I)}/\rho_{(I)}, \qquad c^2_{(I)} \equiv \dot{p}_{(I)}/\dot{\rho}_{(I)}.$$

4. Scalar Perturbations

Perturbations can be separated into scalar, vector and tensor type according to transformation properties w.r.t. the symmetry group and different types do not mix at linear order. It turns out that scalar perturbations is the most difficult part and thus requires much more attention. We proceed with scalar perturbations therefore giving the complete derivation of the corresponding equations in a very general setting.

4.1. Equations

We introduce the spatial scalar harmonics $Y^{(s)}$ defined by an equation

$$\nabla^a \nabla_a Y^{(s)} + k^2 Y^{(s)} = 0,$$

with ∇_a a covariant derivative w.r.t. $g^{(3)}_{ab}$. For $g^{(3)}_{ab} = \delta_{ab}$ we have $\nabla_a = \partial_a$ where ∂_a denotes just an ordinary derivative w.r.t. x^a and $Y^{(s)} = Y^{(s)}_0 e^{\pm i k_a x^a}$. Two more scalar harmonics are defined as

$$Y^{(s)}_a = -\frac{1}{k}\nabla_a Y^{(s)}, \qquad Y^{(s)}_{ab} = \left(\frac{1}{k^2}\nabla_a\nabla_b + \frac{1}{3}g^{(3)}_{ab}\right)Y^{(s)}.$$

We will not use these two latter functions explicitly and note them for a comparison with the literature. $Y^{(s)}$-s describe the Fourier transform for a general spatial metric $g^{(3)}_{ab}$.

Scalar metric perturbations are given by 4 arbitrary scalar functions $\alpha, \beta, \varphi, \gamma$ in the following way

$$ds^2 = a(\eta)^2\Big(-(1+2\alpha)d\eta^2 - 2\partial_a\beta d\eta dx^a + (g^{(3)}_{ab}(1+2\varphi) + 2\nabla_a\nabla_b\gamma)dx^a dx^b\Big), \tag{12}$$

where $\alpha(\eta, x^a) = \alpha(\eta, k)Y^{(s)}(k, x)$ and similar for β, φ and γ. To avoid misunderstanding, recall here we use notations of [39].

The scalar perturbations of the collective stress tensor are

$$T^0_0 = -(\rho+\delta\rho),\ T^0_a = -\frac{1}{k}(\rho+p)\partial_a v,\ T^a_b = (p+\delta p)\delta^a_b + \left(\frac{\nabla^a\nabla_b}{k^2} + \frac{\delta^a_b}{3}\right)\pi^{(s)}, \tag{13}$$

where $\delta\rho(\eta, x^a) = \delta\rho(\eta, k)Y^{(s)}(k, x)$ and similar for $\delta p, v$ and $\pi^{(s)}$. For individual stress tensors $T_{(i)}{}^\mu_\nu$ we introduce similar quantities accompanied with index (i) and the following summation rules hold

$$\begin{aligned} \delta\rho \equiv \delta\rho_{(0)} = \sum_i \delta\rho_{(i)},\ \delta p \equiv \delta p_{(0)} = \sum_i \delta p_{(i)}, \\ (\rho+p)v \equiv (\rho_{(0)} + p_{(0)})v_{(0)} = \sum_i (\rho_{(i)} + p_{(i)})v_{(i)},\ \pi^{(s)} \equiv \pi^{(s)}_{(0)} = \sum_i \pi^{(s)}_{(i)}. \end{aligned} \tag{14}$$

The following notations will be used in the sequel:

$$e_{(I)} \equiv \delta p_{(I)} - c^2_{(I)}\delta\rho_{(I)}, \qquad \delta_{(I)} \equiv \delta\rho_{(I)}/\rho_{(I)}.$$

Constant $w_{(I)}$ results in $e_{(I)} = 0$. Non-zero e_I describes entropic perturbations.

To write down linear perturbation equations we define

$$\chi \equiv a\beta + a^2\dot{\gamma}, \qquad \kappa \equiv 3(-\dot{\varphi} + H\alpha) + \frac{k^2}{a^2}\chi\,. \tag{15}$$

After some algebra one arrives to the following system

$$H\kappa - \frac{k^2}{a^2}\varphi = -4\pi G\delta\rho, \tag{16a}$$

$$-\dot{\varphi} + H\alpha = 4\pi G\frac{a}{k}(\rho + p)v, \tag{16b}$$

$$\dot{\chi} + H\chi - \alpha - \varphi = 8\pi G\frac{a^2}{k^2}\pi^{(s)}, \tag{16c}$$

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = 4\pi G(\delta\rho + 3\delta p), \tag{16d}$$

$$\dot{\delta\rho_{(I)}} + 3H(\delta\rho_{(I)} + \delta p_{(I)}) = (\rho_{(I)} + p_{(I)})\left(\kappa - \frac{k}{a}v_{(I)} - 3H\alpha\right), \tag{16e}$$

$$\frac{\frac{d}{dt}\left(a^4(\rho_{(I)} + p_{(I)})v_{(I)}\right)}{a^4(\rho_{(I)} + p_{(I)})} = \frac{k}{a}\left(\frac{\delta p_{(I)} - \frac{2}{3}\pi^{(s)}_{(I)}}{\rho_{(I)} + p_{(I)}} + \alpha\right), \tag{16f}$$

where we have in top-down direction linear perturbation of: (00) Einstein equation, $(0a)$ Einstein equation, off-diagonal components of the ADM propagator $(G^a_b - \frac{1}{3}\delta^a_b G^c_c)$, Raychaudhuri equation $(G^c_c - G^0_0)$, (0) component of the conservation equation and (a) component of the conservation equation respectively. The above system forms a full set of equations *without* gauge fixing. They are in the so-called *gauge-ready* form. The two last equations can be also written as

$$\dot{\delta}_I + 3H\left(\frac{e_I}{\rho_I} + (c_I^2 - w_I)\delta_I\right) = (1 + w_I)\left(\kappa - \frac{k}{a}v_I - 3H\alpha\right), \tag{17}$$

$$\dot{v}_{(I)} + (1 - 3c^2_{(I)})Hv_{(I)} = \frac{k}{a(1 + w_{(I)})}\left(\frac{e_{(I)}}{\rho_{(I)}} + c^2_{(I)}\delta_{(I)} - \frac{2}{3}\frac{\pi^{(s)}_{(I)}}{\rho_{(I)}} + (1 + w_{(I)})\alpha\right), \tag{18}$$

and

$$\kappa - \frac{k^2}{a^2}\chi = 12\pi G\frac{a}{k}(\rho + p)v, \tag{19}$$

is of use.

Under the gauge transformation $x^\mu \to x^\mu + \xi^\mu$ quantities of interest are transformed as

$$\begin{aligned}&\alpha \to \alpha - \dot{\xi}^t,\ \beta \to \beta - \frac{\xi_t}{a} + a\dot{\xi},\ \varphi \to \varphi - H\xi^t,\ \gamma \to \gamma - \xi,\\ &\chi \to \chi - \xi^t,\ \kappa \to \kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\xi^t,\\ &\delta\rho_{(I)} \to \delta\rho_{(I)} - \dot{\rho}_{(I)}\xi^t,\ v_{(I)} \to v_{(I)} - \frac{k}{a}\xi^t,\ \delta p_{(I)} \to \delta p_{(I)} - \dot{p}_{(I)}\xi^t,\ \pi^{(s)}_{(I)} \to \pi^{(s)}_{(I)},\end{aligned} \tag{20}$$

where $\xi^t = a\xi^\eta$ and $\xi_a = \partial_a\xi$. We see that $\pi^{(s)}_{(I)}$ are gauge invariant and χ is spatially gauge invariant. From the system (16) we see that β and γ enter only in the combination χ resulting in complete spatial gauge invariance. Although one can pick up a spatial gauge this will neither change the form of equations nor simplify the succeeding analysis. For instance, we may choose $\gamma = 0$. Thus we have to fix only the temporal gauge. Notice that fixing α does not fix the temporal gauge completely. Alternatively it may be more convenient to use gauge invariant quantities. In the rest we will stick to the gauge invariant approach.

4.2. Gauge invariant equations for collective quantities

Following the lines of Bardeen's paper [40] we can define gauge invariant quantities

$$v_\chi = v - \frac{k}{a}\chi, \quad \varepsilon = \delta + 3(1+w)H\frac{a}{k}v, \quad \Phi = \alpha - \dot{\chi}, \quad ,\Psi = \varphi - H\chi\,. \tag{21}$$

Then from (16c) one gets

$$-(\Phi + \Psi) = 8\pi G\frac{a^2}{k^2}\pi^{(s)}, \tag{22}$$

and from (16a) and (16b) one has

$$\Psi = 4\pi G\rho\frac{a^2}{k^2}\varepsilon\,. \tag{23}$$

Equation (16f) for $I = 0$, i.e. for collective quantities yields

$$\dot{v}_\chi + Hv_\chi = \frac{k}{a(1+w)}\left(\frac{e}{\rho} + c^2\varepsilon + \Phi(1+w) - \frac{2\pi^{(s)}}{3\rho}\right). \tag{24}$$

Equation (16e) for $I = 0$, i.e. for collective quantities yields

$$\dot{\varepsilon} - 3Hw\varepsilon + \frac{k}{a}(1+w)v_\chi + 2H\frac{\pi^{(s)}}{\rho} = 0. \tag{25}$$

Now one can express v_χ from the latter equation, express Φ through ε and $\pi^{(s)}$ using (22) and (23) and substitute all of this into (24). This will result in a single second order equation for ε and $\pi^{(s)}$. It reads

$$\ddot{\varepsilon}+\dot{\varepsilon}H(2+3c^2-6w)+\varepsilon\left(\dot{H}(1-3w)-15H^2w+9H^2c^2+\frac{k^2}{a^2}c^2\right)= \\ =-\frac{k^2}{a^2}\frac{e}{\rho}-2H\frac{\dot{\pi}^{(s)}}{\rho}+\frac{\pi^{(s)}}{\rho}\left(2H^2(3w-3c^2-2)+\frac{2k^2}{3a^2}\right). \quad (26)$$

One can check this equation against (4.9) in [40]. Our's and Bardeen's are in a perfect agreement with each other. To do this comparison one has to account that dot in Bardeen's paper denotes a derivative w.r.t. the conformal time, our e is equal to $P_0\eta$ in [40] and our $\pi^{(s)}$ is equal to $P_0\pi_T^{(0)}$ in [40]. Taking $\pi^{(s)}=0$ one has

$$\ddot{\varepsilon}+\dot{\varepsilon}H(2+3c^2-6w)+\varepsilon\left(\dot{H}(1-3w)-15H^2w+9H^2c^2+\frac{k^2}{a^2}c^2\right)=-\frac{k^2}{a^2}\frac{e}{\rho}. \quad (27)$$

4.3. Gauge invariant equations for individual quantities

For individual fluids one can define the following gauge invariant quantities:

$$v_{(i)\chi}=v_{(i)}-\frac{k}{a}\chi, \quad \varepsilon_{(i)}=\delta_{(i)}+3(1+w_{(i)})H\frac{a}{k}v_{(i)}. \quad (28)$$

Then for i-th fluid an analog of equation (24) becomes

$$\dot{v_{(i)\chi}}+Hv_{(i)\chi}=\frac{k}{a(1+w_{(i)})}\left(\frac{e_{(i)}}{\rho_{(i)}}+c_{(i)}^2\varepsilon_{(i)}+\Phi(1+w_{(i)})-\frac{2\pi_{(i)}^{(s)}}{3\rho_{(i)}}\right). \quad (29)$$

We note the only change is that everything that can carry a fluid index (i) has acquired it. An analog of equation (25) is not so straightforward and is given by

$$\dot{\varepsilon}_{(i)}-3Hw_i\varepsilon_{(i)}+\frac{k}{a}(1+w_{(i)})v_{(i)\chi}(1-h)+2H\frac{\pi_{(i)}^{(s)}}{\rho_{(i)}}=-\frac{k}{a}(1+w_{(i)})hv_\chi. \quad (30)$$

where $h = 3\dot{H}\frac{a^2}{k^2}$. Further manipulations bring us to a system of equations for $\varepsilon_{(i)}$

$$\begin{aligned}
\ddot{\varepsilon}_{(i)} + \dot{\varepsilon}_{(i)} H \left(2 + 3c_{(i)}^2 - 6w_{(i)} + \frac{\dot{h}}{H(1-h)}\right) + \\
+ \varepsilon_{(i)} \left(-3\dot{H}(c_{(i)}^2 + w_{(i)}) + 9H^2 c_{(i)}^2 - 15H^2 w_{(i)} - 3Hw_{(i)} \frac{\dot{h}}{1-h} + \frac{k^2}{a^2} c_{(i)}^2\right) = \\
= -\frac{k^2}{a^2}\frac{e_{(i)}}{\rho_{(i)}} + 3\dot{H}\left(\frac{e_{(i)}}{\rho_{(i)}} - \frac{1+w_{(i)}}{1+w}\frac{e}{\rho}\right) + \dot{\varepsilon}\frac{1+w_{(i)}}{1+w}\frac{\dot{h}}{1-h} - \\
- \varepsilon\frac{1+w_{(i)}}{1+w}\left(3Hw\frac{\dot{h}}{1-h} + \dot{H}(1+3c^2)\right) + \\
+ \frac{2k^2}{3a^2}\frac{\pi_{(i)}^{(s)}}{\rho_{(i)}} - 2\frac{1+w_{(i)}}{a^2 H}\frac{d}{dt}\left(\frac{a^2 H^2 \pi_{(i)}^{(s)}}{\rho_{(i)}(1+w_{(i)})}\right) - \\
- 2H\frac{\dot{h}}{1-h}\left(\frac{\pi_{(i)}^{(s)}}{\rho_{(i)}} - \frac{\pi^{(s)}}{\rho}\frac{1+w_{(i)}}{1+w}\right). \qquad (31)
\end{aligned}$$

Here we note an appearance of a number of singular terms with a common prefactor $\frac{\dot{h}}{1-h}$. Since the above equation is manifestly gauge invariant, one cannot remove this singularity by means of gauge freedom.

Further one can eliminate collective quantities by virtue of summation rules (14) and the preceding analysis. The resulting equation is

$$\begin{aligned}
\ddot{\varepsilon}_{(i)} + \dot{\varepsilon}_{(i)} H \left(2 + 3c_{(i)}^2 - 6w_{(i)}\right) + \\
+ \varepsilon_{(i)} \left(-3\dot{H}(c_{(i)}^2 + w_{(i)}) + 9H^2 c_{(i)}^2 - 15H^2 w_{(i)} + \frac{k^2}{a^2} c_{(i)}^2\right) = \\
= -\frac{k^2}{a^2}\frac{e_{(i)}}{\rho_{(i)}} + \frac{12\pi G}{\rho_{(i)}}\sum_k \left((\rho_{(i)} + p_{(i)})e_{(k)} - (\rho_{(k)} + p_{(k)})e_{(i)}\right) + \\
+ 4\pi G(1+w_{(i)})\sum_k \rho_{(k)}\varepsilon_{(k)}(1+3c_{(k)}^2) + \qquad (32) \\
+ \frac{12\pi G H}{3\dot{H} - \frac{k^2}{a^2}}\sum_k \Big[\rho_{(k)}(1+3c_{(k)}^2)\left((1+w_{(k)})(\dot{\varepsilon}_{(i)} - 3Hw_{(i)}\varepsilon_{(i)}\right) - \\
- (1+w_{(i)})(\dot{\varepsilon}_{(k)} - 3Hw_{(k)}\varepsilon_{(k)})\big)\Big] +
\end{aligned}$$

$$+\frac{2k^2}{3a^2}\frac{\pi_{(i)}^{(s)}}{\rho_{(i)}} - 2\frac{1+w_{(i)}}{a^2H}\frac{d}{dt}\left(\frac{a^2H^2\pi_{(i)}^{(s)}}{\rho_{(i)}(1+w_{(i)})}\right) +$$
$$+\frac{24\pi GH^2}{3\dot{H}-\frac{k^2}{a^2}}\sum_k \rho_{(k)}(1+3c_{(k)}^2)\left((1+w_{(k)})\frac{\pi_{(i)}^{(s)}}{\rho_{(i)}} - (1+w_{(i)})\frac{\pi_{(k)}^{(s)}}{\rho_{(k)}}\right).$$

5. Perturbations in the non-local model

5.1. Bianchi identity

We know that the Einstein equations are constrained with the Bianchi identity which says $\nabla_\mu G^\mu_\nu \equiv 0$. In our case we have more than just Einstein-Hilbert Lagrangian but all the additional ingredients we have are covariant terms. This guarantees that the Bianchi identity holds trivially without imposing any extra condition. On the other hand this implies thanks to arbitrariness of coefficients f_{I_n} that each separate term does covariantly conserve. Indeed, each f_{I_n} is a coefficient in front of some covariant structure in the Einstein equations, say τ^μ_ν. Assuming that only one of f-s coefficients is non-zero we come to a conclusion that the corresponding structure must covariantly conserve due to Bianchi identities, i.e. $\nabla_\mu \tau^\mu_\nu \equiv 0$. In other words it resembles a conserving perfect fluid stress-energy tensor. The same argument is applicable to all the f-s coefficients as well as their arbitrary combinations.

The above arguments imply that thanks to Bianchi identity the parts with different $\mathcal{F}_I$ covariantly conserve separately. To make use of this we define

$$\begin{aligned}
T_0{}^\mu_\nu &= T^\mu_\nu,\\
T_1{}^\mu_\nu &= -2\lambda\tilde{G}^\mu_\nu\tilde{\mathcal{F}}_1(\Box)R + 2\lambda(\nabla^\mu\partial_\nu - \delta^\mu_\nu\Box)\tilde{\mathcal{F}}_1(\Box)R +\\
&\quad + \lambda\mathcal{L}_1{}^\mu_\nu - \frac{\lambda}{2}\delta^\mu_\nu\left(\mathcal{L}_1{}^\sigma_\sigma + \bar{\mathcal{L}}_1\right),\\
T_2{}^\mu_\nu &= -\frac{1}{2}\,\lambda R\mathcal{F}_2(\Box)\tilde{G}^\mu_\nu - 2\lambda\tilde{G}^\mu_\beta\mathcal{F}_2(\Box)\tilde{G}^\beta_\nu + \frac{\lambda}{2}\delta^\mu_\nu\tilde{G}^\alpha_\beta\mathcal{F}_2(\Box)\tilde{G}^\beta_\alpha +\\
&\quad + 2\lambda\left(\nabla_\rho\nabla_\nu\mathcal{F}_2(\Box)\tilde{G}^{\mu\rho} - \frac{1}{2}\Box\mathcal{F}_2(\Box)\tilde{G}^\mu_\nu - \frac{1}{2}\delta^\mu_\nu\nabla_\sigma\nabla_\rho\mathcal{F}_2(\Box)\tilde{G}^{\sigma\rho}\right) +\\
&\quad + \lambda\mathcal{L}_2{}^\mu_\nu - \frac{\lambda}{2}\delta^\mu_\nu\left(\mathcal{L}_2{}^\sigma_\sigma + \bar{\mathcal{L}}_2\right) + 2\lambda\tilde{\Delta}^\mu_\nu,\\
T_4{}^\mu_\nu &= 2\lambda\left(R_{\alpha\beta} + 2\nabla_\alpha\nabla_\beta\right)\mathcal{F}_4(\Box)C_\nu{}^{\alpha\beta\mu}.
\end{aligned}$$

Now Einstein equations can be written in an extremely concise form

$$M_P^2 G^\mu_\nu = \sum_I T_I{}^\mu_\nu - \Lambda\delta^\mu_\nu \qquad (33)$$

and moreover we have

$$\nabla_\mu T_I{}^\mu_\nu = 0 \text{ for any } I.$$

One recognizes here the system of minimally coupled perfect fluids minimally coupled to gravity.

5.2. Final step

The next step is to split each T_I as

$$T_I = \sum_{n\geq 0} f_{In} T_{In}\,, \tag{34}$$

keeping in mind that each component is covariantly conserved on its own. This is not enough though. since this produces infinitely many perturbation functions. What is good however, for a given I all the components T_{In} are related. One can show by a straightforward computation that

$$\delta T_{In+1} = (\Box + 8H^2)\delta T_{In}\,. \tag{35}$$

This is the most crucial formula in all this study since it allows to write all the perturbation equations in a closed finite form without introducing infinitely many unknown functions.

So, schematically, we go from the tensorial form of the last equation to its components, then we substitute the result in all equations (32) and derive therefore four equations for $\varepsilon_{0,1,2,4}$ coupled one to each other. This accomplishes the task of deriving these equations. The next step of studying them is still very difficult but we hope it is possible at least numerically.

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