Some Ansätze in Nonlocal Modified Gravity

Ivan Dimitrijević[*]
Faculty of Mathematics, University of Belgrade
Studentski Trg 16, 11000 Belgrade, SERBIA

Abstract

Equations of motion in the nonlocal modifications of gravity are usually nonlinear differential equations of an infinite order. A solution of such equations is mainly hard to obtain and general solutions are almost unknown. Thus, we are limited to some particular solutions. In this paper we present some ansätze that can help find some solutions in the certain suitable forms.

1. Introduction

Attempts to modify General Relativity started already a few years after its beginning and they were mainly motivated by purely mathematical reasons. The discovery of the accelerated expansion of the Universe in 1998 inspired a lot of new research in the field. In this paper, we consider nonlocal modification (for a review see [1, 2]) of gravity with no matter defined by action

$$S = \int \left(\frac{R - 2\Lambda}{16\pi G} + C\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R) \right) \sqrt{-g}\, d^4x, \tag{1}$$

where $\Box = \nabla_\mu \nabla^\mu$ is the D'Alembert operator related to metric tensor $g_{\mu\nu}$, $\mathcal{F}$ is an analytic function of $\Box$ operator and $\mathcal{G}$ and $\mathcal{H}$ are differentiable functions of the scalar curvature. Varying the action over metric tensor $g_{\mu\nu}$ one can obtain equations of motion (see [7] for details).

$$\begin{aligned} &C\Big(-\frac{1}{2} g_{\mu\nu}\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R) + (R_{\mu\nu}\Phi - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\Phi) \\ &+ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \big(g_{\mu\nu} g^{\alpha\beta}\partial_\alpha \Box^l \mathcal{H}(R)\partial_\beta \Box^{n-1-l}\mathcal{G}(R) \\ &- 2\partial_\mu \Box^l \mathcal{H}(R)\partial_\nu \Box^{n-1-l}\mathcal{G}(R) + g_{\mu\nu}\Box^l\mathcal{H}(R)\Box^{n-l}\mathcal{G}(R)\big)\Big) = -\frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G}, \end{aligned} \tag{2}$$

[*] e-mail address: ivand@matf.bg.ac.rs

where Φ is given by

$$\Phi = \mathcal{H}'(R)\mathcal{F}(\Box)\mathcal{G}(R) + \mathcal{G}'(R)\mathcal{F}(\Box)\mathcal{H}(R) \tag{3}$$

and $'$ denotes derivatives over R.

These equations can be of arbitrary high order and some ansätze are frequently used to solve them.

We will discuss 4 ansätze:

- $\Box R = rR + s$
- $\Box R = qR^2$
- $\Box R = qR^3$
- $\Box^n R = c_n R^{An+B}$ for all $n \geq 1$, A and B are some constants.

These ansätze can considerably simplify solving equations of motion, but still we have to find some solutions satisfying them before their application to the equations of motion. That task can also be a challenging one even in the simplest case. Linear ansatz, for example, is a fourth order nonlinear differential equation of cosmological scale factor $a(t)$. Thus, in the sequel we will be looking for the solution of the particular ansatz in the suitably chosen form.

We assume that Friedmann-Lemaître-Robertson-Walker (FLRW) metric $ds^2 = -dt^2 + a^2(t)\big(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\big)$ is used. All three possibilities for curvature parameter k $(0, \pm 1)$ are investigated. Scalar curvature in this context is given by

$$R = 6\Big(\frac{\ddot{a}}{a} + \big(\frac{\dot{a}}{a}\big)^2 + \frac{k}{a^2}\Big). \tag{4}$$

These ansätze and their solutions have been used in several papers to get various cosmological solutions [2, 3, 4, 5].

2. Linear ansatz: $\Box R = rR + s$

To begin with, we search for the solution of the ansatz

$$\Box R = rR + s, \tag{5}$$

for some real constants r and s in the form

$$a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t}). \tag{6}$$

Direct calculation gives the corresponding expressions for the Hubble parameter and scalar curvature

$$\begin{aligned} H(t) &= \frac{\lambda(\sigma e^{\lambda t} - \tau e^{-\lambda t})}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}, \\ R(t) &= \frac{6\left(2a_0^2\lambda^2\left(\sigma^2 e^{4t\lambda} + \tau^2\right) + ke^{2t\lambda}\right)}{a_0^2\left(\sigma e^{2t\lambda} + \tau\right)^2}, \end{aligned} \tag{7}$$

Moreover, we can calculate $\Box R$ and plug it into ansatz (5) and get

$$\begin{aligned} -12\lambda^2 e^{2t\lambda}\left(4a_0^2\lambda^2\sigma\tau - k\right) = 6r\left(2a_0^2\lambda^2\left(\sigma^2 e^{4t\lambda} + \tau^2\right) + ke^{2t\lambda}\right) \\ + sa_0^2\left(\sigma e^{2t\lambda} + \tau\right)^2. \end{aligned} \tag{8}$$

Last equation can be interpreted as a polynomial in $e^{2\lambda t}$. Thus, it can be split into the following system of linear equations in r and s

$$\begin{aligned} a_0^2\tau^2\left(12r\lambda^2 + s\right) = 0, \\ 2\left(a_0^2 s\sigma\tau + 24a_0^2\lambda^4\sigma\tau + 3kr - 6k\lambda^2\right) = 0, \\ a_0^2\sigma^2\left(12r\lambda^2 + s\right) = 0. \end{aligned} \tag{9}$$

It has two solutions

$$r = 2\lambda^2,\ s = -24\lambda^4, \tag{10}$$

and

$$s = -12r\lambda^2,\ k = 4a_0^2\lambda^2\sigma\tau. \tag{11}$$

Therefore, the solution of the form (6) is a solution of the ansatz (5) for any value of real parameters τ, σ, a_0 and λ if we set $r = 2\lambda^2$ and $s = -24\lambda^4$. Note that in all these cases $r > 0$ and $s < 0$. Alternatively, assuming $k = 4a_0^2\lambda^2\sigma\tau$ solution (6) satisfies the ansatz (5) for any r and $s = -12r\lambda^2$. Note that in this case we have constant scalar curvature $R = 12\lambda^2$.

Secondly, we can look for a solution in the form

$$\begin{aligned} a(t) &= a_0 e^{\frac{\lambda}{2}t^2}, \\ H(t) &= \lambda t, \\ R(t) &= 6a_0^{-2}e^{-t^2\lambda}\left(2a_0^2t^2\lambda^2e^{t^2\lambda} + a_0^2\lambda e^{t^2\lambda} + k\right). \end{aligned} \tag{12}$$

Using the same method as in the previous case the ansatz (5) gives

$$\begin{aligned} 12a_0^2rt^2\lambda^2e^{t^2\lambda} + 6a_0^2r\lambda e^{t^2\lambda} + a_0^2se^{t^2\lambda} + 72a_0^2t^2\lambda^3e^{t^2\lambda} \\ + 24a_0^2\lambda^2e^{t^2\lambda} + 6kr - 12kt^2\lambda^2 - 12k\lambda = 0. \end{aligned} \tag{13}$$

Hence, we interpret this as a two variable polynomial in t^2 and $e^{\lambda t^2}$ to split it into system of equations corresponding to each coefficient

$$\begin{aligned} 6k(r - 2\lambda) = 0, \\ a_0^2\left(6r\lambda + s + 24\lambda^2\right) = 0, \\ -12k\lambda^2 = 0, \\ 12a_0^2\lambda^2(r + 6\lambda) = 0. \end{aligned} \tag{14}$$

To get a nonconstant solution of $a(t)$ in (12) the third equation in the system (14) requires $k = 0$. With this additional assumption we can solve (14) and get

$$r = -6\lambda, \quad s = 12\lambda^2. \tag{15}$$

Now, in (12) we have solutions for all values of λ and a_0. In the contrast to the first case we have $s > 0$.

To summarize, we obtained two families of solutions:

1. $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$ satisfying ansatz $\Box R = 2\lambda^2 R - 24\lambda^4$ and
2. $a(t) = a_0 e^{\frac{\lambda}{2}t^2}$ satisfying ansatz $\Box R = -6\lambda R + 12\lambda^2$.

Using this method one can also get solutions in the form $a(t) = a_0\sqrt{|t|}e^{\frac{\lambda}{2}t^2}$ and $a(t) = a_0\sqrt{\sigma e^{\lambda t} + \tau e^{-\lambda t}}$. These solutions are obtained only in the flat metric case ($k = 0$) [8].

3. Quadratic ansatz: $\Box R = qR^2$

We look for the solutions in the form

$$a(t) = a_0|t - d_1|^\alpha. \tag{16}$$

The first consequences are

$$\begin{aligned}
H(t) &= \alpha(t - d_1)^{-1},\\
R(t) &= 6(\alpha(2\alpha - 1)(t - d_1)^{-2} + \frac{k}{a_0^2}(t - d_1)^{-2\alpha}),\\
\Box R &= 12\alpha(\alpha - 1)\Big(3(2\alpha - 1)(t - d_1)^{-4} + \frac{k}{a_0^2}(t - d_1)^{-2\alpha-2}\Big).
\end{aligned} \tag{17}$$

Thus, the ansatz becomes

$$\begin{aligned}
&\alpha(\alpha - 1)\Big(3(2\alpha - 1)(t - d_1)^{-4} + \frac{k}{a_0^2}(t - d_1)^{-2\alpha-2}\Big)\\
&\qquad = 3q\Big(\alpha(2\alpha - 1)(t - d_1)^{-2} + \frac{k}{a_0^2}(t - d_1)^{-2\alpha}\Big)^2.
\end{aligned} \tag{18}$$

We have to solve equation (18) for arbitrary t and therefore we will split it into 2 cases. At first, let us analyze the problem for $k = 0$. The equation (18) becomes

$$\alpha(\alpha - 1)(2\alpha - 1)(t - d_1)^{-4} = q\alpha^2(2\alpha - 1)^2(t - d_1)^{-4}. \tag{19}$$

There are 2 obvious solutions of the last equation, namely $\alpha = 0$ and $\alpha = \frac{1}{2}$, for any value of parameter q. Note that these two solutions fulfill condition $R = 0$.

When $\alpha \neq 0$ and $\alpha \neq \frac{1}{2}$ then $R \neq 0$ and equation (19) becomes

$$\alpha - 1 = q\alpha(2\alpha - 1), \tag{20}$$

and can be solved for any α with suitably chosen value of q. Thus, for

$$q_\alpha = \frac{\alpha - 1}{\alpha(2\alpha - 1)}, \tag{21}$$

we have the following solution of the ansatz $\Box R = q_\alpha R^2$

$$a(t) = a_0|t - d_1|^\alpha, \quad \alpha \neq 0, \alpha \neq \frac{1}{2}. \tag{22}$$

The second possibility is that k is nonzero and equation (18) can be rewritten as ($\tau = t - d_1$)

$$\begin{aligned} &\alpha(\alpha - 1)\Big(3(2\alpha - 1)\tau^{-4} + \frac{k}{a_0^2}\tau^{-2\alpha-2}\Big) \\ &= 3q\Big(\alpha^2(2\alpha - 1)^2\tau^{-4} + 2\alpha(2\alpha - 1)\frac{k}{a_0^2}\tau^{-2\alpha-2} + \frac{k^2}{a_0^4}\tau^{-4\alpha}\Big). \end{aligned} \tag{23}$$

The coefficient in front of $(t - d_1)^{-4\alpha}$ is $P = \frac{3qk^2}{a_0^4}$. If $P \neq 0$, since we are looking for solutions independent of time, we have to require $-4\alpha = -4$ or $-4\alpha = -2\alpha - 2$. Both of these alternatives lead to $\alpha = 1$ and the equation (23) is simplified to

$$0 = 3q\Big(1 + \frac{k}{a_0^2}\Big)^2(t - d_1)^{-4}. \tag{24}$$

Since $q = 0$ would imply $P = 0$ we have only one solution for $k = -1$ and $q \neq 0$

$$a(t) = |t - d_1|. \tag{25}$$

This solution also satisfies $R = 0$. Last possibility is $P = 0$, or in other words $q = 0$. In this case ansatz becomes $\Box R = 0$ and equation (18) is transformed to

$$\alpha(\alpha - 1)\Big(3(2\alpha - 1)(t - d_1)^{-4} + \frac{k}{a_0^2}(t - d_1)^{-2\alpha-2}\Big) = 0. \tag{26}$$

It has two solutions $\alpha = 1$ and $\alpha = 0$ or in terms of a scale factor $a(t) = a_0|t - d_1|$ and $a(t) = a_0$.

Quadratic ansatz gives the solution (16) in six cases:

1. $k = 0$, $\alpha = 0$, $q \in \mathbb{R}$,
2. $k = 0$, $\alpha = \frac{1}{2}$, $q \in \mathbb{R}$,
3. $k = 0$, $\alpha \neq 0$ and $\alpha \neq \frac{1}{2}$, $q_\alpha = \frac{\alpha-1}{\alpha(2\alpha-1)}$,
4. $k = -1$, $\alpha = 1$, $q \neq 0$, $a_0 = 1$,
5. $k \neq 0$, $\alpha = 0$, $q = 0$,
6. $k \neq 0$, $\alpha = 1$, $q = 0$.

4. Cubic ansatz: $\Box R = qR^3$

We again look for a solution of the form (16). Using (16) and (17) the ansatz is transformed to

$$\begin{aligned}\alpha(\alpha-1)\Big(3(2\alpha-1)(t-d_1)^{-4}+\frac{k}{a_0^2}(t-d_1)^{-2\alpha-2}\Big)&\\ =18q\Big(\alpha(2\alpha-1)(t-d_1)^{-2}+\frac{k}{a_0^2}(t-d_1)^{-2\alpha}\Big)^3.&\end{aligned} \tag{27}$$

In a flat model ($k=0$) the equation is simplified considerably and reads

$$\alpha(\alpha-1)(2\alpha-1)(t-d_1)^{-4}=6q\alpha^3(2\alpha-1)^3(t-d_1)^{-6}. \tag{28}$$

Since it has to be satisfied for all t, both sides of the equation have to be zero. We have 3 cases

1. $\alpha=0$,
2. $\alpha=\frac{1}{2}$,
3. $\alpha=1$ and $q=0$.

If $k=\pm1$ the equation (27) reads

$$\begin{aligned}&\alpha(1-\alpha)(2\alpha-1)(t-d_1)^{-4}+6q\alpha^3(2\alpha-1)^3(t-d_1)^{-6}+\frac{6kq}{a_0^6}(t-d_1)^{-6\alpha}\\ &+\frac{18q}{a_0^4}\alpha(2\alpha-1)(t-d_1)^{-2-4\alpha}+\frac{k}{3a_0^2}\alpha(1-\alpha)(t-d_1)^{-2-2\alpha}\\ &+\frac{18qk}{a_0^2}\alpha^2(2\alpha-1)^2(t-d_1)^{-4-2\alpha}=0.\end{aligned} \tag{29}$$

In the equation (29) there are at most six different powers of $t-d_1$. They are all different, and therefore no two of them can be combined unless $\alpha\in\{0,\frac{1}{2},\frac{2}{3},1,2\}$. Hence, when $\alpha\notin\{0,\frac{1}{2},\frac{2}{3},1,2\}$ we have to satisfy the following system

$$\begin{aligned}\alpha(1-\alpha)(2\alpha-1)&=0,\\ 6q\alpha^3(2\alpha-1)^3&=0,\\ \frac{6kq}{a_0^6}&=0,\\ \frac{18q}{a_0^4}\alpha(2\alpha-1)&=0,\\ \frac{k}{3a_0^2}\alpha(1-\alpha)&=0,\\ \frac{18qk}{a_0^2}\alpha^2(2\alpha-1)^2&=0.\end{aligned} \tag{30}$$

The first equation requires that $\alpha \in \{0, \frac{1}{2}, 1\}$, but all these values are excluded so the system has no solutions. It remains to check if there is a solution of (29) in the set $\{0, \frac{1}{2}, \frac{2}{3}, 1, 2\}$. For $\alpha = 0$ it reads $\frac{6qk}{a_0^6} = 0$ and is satisfied if $q = 0$. For $\alpha = \frac{1}{2}$ we obtain

$$-\frac{k}{4a_0^2}(t-d_1)^{-3} = \frac{18qk}{a_0^6}(t-d_1)^{-3}. \tag{31}$$

In other words we have $q = -\frac{a_0^4}{72}$. When $\alpha = \frac{2}{3}$ we have

$$(t-d_1)^{-4} + \frac{k}{a_0^2}(t-d_1)^{-\frac{10}{3}} = -81q\Big(\frac{2}{9}(t-d_1)^{-2} + \frac{k}{a_0^2}(t-d_1)^{-\frac{4}{3}}\Big)^3. \tag{32}$$

This equation has no solution that would hold for all values of t. For $\alpha = 1$ we obtain

$$18q\Big(1 + \frac{k}{a_0^2}\Big)^3(t-d_1)^{-6} = 0. \tag{33}$$

In this case we have two solutions. The first is $q = 0$ for both values of $k = \pm 1$, and the second $k = -1$ and $a_0 = 1$.

The last case $\alpha = 2$ yields the following equation

$$9(t-d_1)^{-4} + \frac{k}{a_0^2}(t-d_1)^{-6} = 9q\Big(6(t-d_1)^{-2} + \frac{k}{a_0^2}(t-d_1)^{-4}\Big)^3. \tag{34}$$

This equation does not have any solution.

Cubic ansatz gives the solution (16) in seven cases:

1. $k = 0$, $\alpha = 0$, $q \in \mathbb{R}$,
2. $k = 0$, $\alpha = \frac{1}{2}$, $q \in \mathbb{R}$,
3. $k = 0$, $\alpha = 1$, $q = 0$,
4. $k \neq 0$, $\alpha = 0$, $q = 0$,
5. $k \neq 0$, $\alpha = \frac{1}{2}$, $q = -\frac{a_0^4}{72}$,
6. $k \neq 0$, $\alpha = 1$, $q = 0$,
7. $k = -1$, $\alpha = 1$, $q \neq 0$, $a_0 = 1$.

5. Ansatz $\Box^n R = c_n R^{An+B}$

We consider another ansatz of the form $\Box^n R = c_n R^{An+B}$, where A and B are real constants, and $n \in \mathbb{N}$. Since we have an infinite number of conditions on the scalar curvature we have first to be sure that two consecutive conditions are consistent. Calculation of $\Box^{n+1}R$ in two ways, using the formula $\Box R^p = pR^{p-1}\Box R - p(p-1)R^{p-2}\dot{R}^2$ yields

$$\begin{aligned}
&\Box^{n+1}R = \Box c_n R^{An+B} \\
&= c_n((An+B)R^{An+B-1}\Box R - (An+B)(An+B-1)R^{An+B-2}\dot{R}^2) \\
&= c_n(An+B)(c_1 R^{An+A+2B-1} - (An+B-1)R^{An+B-2}\dot{R}^2) \\
&= c_{n+1}R^{An+A+B}.
\end{aligned} \tag{35}$$

$$An + A + 2B - 1 = An + A + B, \tag{36}$$

$$\dot{R}^2 = R^{A+B+1}, \tag{37}$$

$$c_{n+1} = c_n(An+B)(c_1 - An - B + 1). \tag{38}$$

Equation (36) implies that $B = 1$. Sequence c_n is defined by (38) which can be solved to obtain

$$c_n = c_1 \prod_{k=1}^{n-1}(Ak+1)(c_1 - Ak), \tag{39}$$

where c_1 is the first element of the sequence and will be determined later. General solution of equation (37) is of the form

$$R(t) = \left(\frac{2}{At - d_1}\right)^{\frac{2}{A}}, \tag{40}$$

with an arbitrary constant d_1. In order to have a solution of the form given in the last equation we have to check the initial condition $\Box R = c_1 R^{A+1}$. Using the solution (40) we can get expression for the Hubble parameter and scale factor

$$\begin{aligned}
H(t) &= \frac{2c_1 + 2 + A}{3(At - d_1)}, \\
a(t) &= a_0|At - d_1|^{\frac{2c_1+2+A}{3A}}, \qquad a_0 > 0.
\end{aligned} \tag{41}$$

On the other hand scale factor must fulfill (40), ie.

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \left(\frac{2}{At - d_1}\right)^{\frac{2}{A}}. \tag{42}$$

Equation (42) becomes

$$\begin{aligned}
a_0^2\frac{(4c_1 + 4 + 2A)(4c_1 + 4 - A)}{9}&(At - d_1)^{\frac{4(c_1 - A + 1)}{3A}} + 2k \\
&= \frac{1}{3}4^{\frac{1}{A}}a_0^2(At - d_1)^{\frac{4c_1+2A-2}{3A}}.
\end{aligned} \tag{43}$$

If we require that all three terms in the above equation have the same degree in $At - d_1$ it gives the following conditions

$$\begin{aligned} \frac{4(c_1 - A + 1)}{3A} &= \frac{4c_1 + 2A - 2}{3A} = 0, \\ a_0^2 \frac{(4c_1 + 4 + 2A)(4c_1 + 4 - A)}{9} + 2k &= \frac{1}{3} 4^{\frac{1}{A}} a_0^2. \end{aligned} \tag{44}$$

We see that $A = 1$, $c_1 = 0$ and a_0 is determined by $a_0^2 = -3k$. In the case $k = 1$ obviously there is no solution, in the flat case $k = 0$ we have only the solution $a_0 = 0$ and hence $a(t) = 0$. The case $k = -1$ is the most interesting and it yields solution

$$k = -1, \qquad a(t) = \sqrt{3}|t - d_1|. \tag{45}$$

Another possibility is $\frac{4(c_1 - A+1)}{3A} = \frac{4c_1+2A-2}{3A} \neq 0$. Therefore we have $A = 1$, $c_1 \neq 0$ and the remaining condition is

$$a_0^2 \frac{(4c_1 + 6)(4c_1 + 3)}{9} (t - d_1)^{\frac{4c_1}{3}} + 2k = \frac{4}{3} a_0^2 (t - d_1)^{\frac{4c_1}{3}}. \tag{46}$$

It holds when $k = 0$ and $8c_1^2 + 18c_1 + 3 = 0$. The corresponding solutions are

$$k = 0, \qquad a(t) = a_0 |t - d_1|^{\frac{2c_1+3}{3}}, \qquad c_1 = \frac{-9 \pm \sqrt{57}}{8}. \tag{47}$$

Now suppose $\frac{4c_1+2A-2}{3A} = 0 \neq \frac{4(c_1-A+1)}{3A}$. Hence equation (43) is transformed into

$$\begin{aligned} c_1 &= \frac{1 - A}{2}, \\ -\frac{1}{3} 4^{\frac{1}{A}} a_0^2 + 2k &= 0, \\ 2a_0^2 (2 - A) &= 0. \end{aligned} \tag{48}$$

Obviously, we have $A = 2$ and $c_1 = -\frac{1}{2}$, hence in the same way as previous case we obtain nontrivial solution only for $k = 1$ and $a_0 = \sqrt{3}$:

$$k = 1, \qquad a(t) = \sqrt{3}\sqrt{|2t - d_1|} \qquad c_1 = -\frac{1}{2}. \tag{49}$$

The remaining cases $\frac{4(c_1-A+1)}{3A} = 0$, $\frac{4c_1+2A-2}{3A} \neq 0$ and $\frac{4(c_1-A+1)}{3A} \neq 0$, $\frac{4c_1+2A-2}{3A} \neq 0$ lead to $a_0 = 0$ and hence trivial solution $a(t) = 0$.

We obtained three solutions:

1. $A = 1$, $c_1 = 0$, $k = -1$, $a(t) = \sqrt{3}|t - d_1|$,
2. $A = 1$, $c_1 = \frac{-9 \pm \sqrt{57}}{8}$, $k = 0$, $a(t) = a_0 |t - d_1|^{\frac{2c_1+3}{3}}$,
3. $A = 2$, $c_1 = -\frac{1}{2}$, $k = 1$, $a(t) = \sqrt{3|2t - d_1|}$.

6. Conclusion

In this paper we investigated some ansätze for the class of nonlocal gravity models given by the action in the form (1). Since the equations of motion that are obtained in this setting can be very difficult to solve we explored some ansätze that are useful to simplify the problem. Also, we obtained few families of solutions for these ansätze. The remaining task is to verify if these solutions also satisfy equations of motion (2). This question is partially addressed in the several papers [3, 4, 5, 6].

Acknowledgements

I would like to thank Branko Dragovich, Zoran Rakić and Jelena Grujić for the inspirative discussions. This investigation is supported by Ministry of Education, Science and Technological Development of the Republic of Serbia, grant No 174012.

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