String-like structures in the real and complex Kerr geometry *

Alexander Burinskii†
NSI Russian Acad. Sciences

Abstract

The 4d Kerr geometry displays many wonderful relations with quantum world and, in particular, with superstring theory. The lightlike structure of fields near the Kerr singular ring is similar to the structure of Sen solution for a closed heterotic string. Another string, open and complex, appears in the initiated by Newman complex representation of the Kerr geometry. Combination of these strings forms a membrane source of the Kerr geometry which is parallel to the string/M-theory unification. In this paper we give one more evidence of this relationship, emergence of the Calabi-Yau twofold (K3 surface) in twistorial structure of the Kerr geometry as a consequence of the Kerr theorem. Finally, we indicate that the Kerr stringy system may correspond to a complex embedding of the critical N=2 superstring.

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1. Introduction

It is commonly recognized now that black holes (BH) are akin to elementary particles. On the other hand, the gravity and BH's represent now important constituents of superstring theory. However, in spite of these close relationships, the consistent unification of the gravity, strings and elementary particles is not reached. To understand what prevents from such unification, one should analyze all the consistent points and contradictions. The Kerr solution plays in this respect especial role, since it represents a metric of the rotating BH or a classical "spinning mass" [1] which displays a consistency with gravitational field of elementary particles. Angular momentum of the Kerr solution is $J = m|a|$, where parameter $a = J/m$ is radius of the Kerr singular ring. The charged Kerr-Newman (KN) solution has gyromagnetic ratio $g = 2$, as that of the Dirac electron [2, 3], and therefore, the KN gravitational field corresponds to the background metric of the electron with great precision, indicating that the conflict between

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† e-mail address: bur@ibrae.ac.ru

gravity and the spinning elementary particles is not impassable. The spin of electron is extreme high, and the ratio a/m (in the dimensionless units $c = G = \hbar = 1$) is about 10^{22}. For $|a| < m$, the Kerr singular ring is covered by the BH horizon, however, for parameters of the elementary particles $|a| >> m$, the black hole horizons disappear, and the Kerr singular ring turns out to be naked, taking the Compton radius $a = \hbar/2m$, contrary to the pointlike structureless electron. Also, instead of the expected week gravitational field and the consistent with quantum theory flat background in vicinity of the electron core, the KN gravitational field exhibits a singular ring of the Compton size. Fortunately, this trouble may be regulated by a special procedure – introduction of some source covering the Kerr singular ring. Structure of such a source was specified by many physicists step by step during more than four decades, in particular in the papers [4, 5, 6, 7, 8, 9, 10, 11]. As a result, the consistent regular source of the KN solution, creating the necessary very weak gravitational field, acquired the form of a highly oblate bubble (rotating membrane) bounded by a closed relativistic string. Along with this closed string,[11, 12], an open complex string was obtained in the complex structure of the Kerr-Schild (KS) geometry [13]. These two strings form together a regular membrane source, which is parallel with the regular enhancon model [14] used in the superstring/M-theory unification [15]. Finally, it has been obtained recently, [16], that the twistorial structure of the four dimensional KS geometry, being combined with the orientifold structure of the complex Kerr string, creates the Calabi-Yau twofold (K3 surface) on the projective twistor space determined by the Kerr theorem. It confirms close relationships of the KS geometry with the basic structures of the superstring theory and indicates presence of some underlying theory which unifies the Kerr gravity with physics of elementary particles and superstring theory. Origin of this unification may lie in the complex N=2 critical superstring [17] which, like the KS geometry, has inherent twistorial structure [18, 19] and may consistently be embedded in the *complex* four-dimensional KS geometry.

2. Real structure of the KN geometry

KN metric is represented in the Kerr-Schild (KS) form [3],

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski background in Cartesian coordinates $\mathrm{x} = x^\mu = (t, x, y, z)$,

$$h = P^2 \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}, \quad P = (1 + Y\bar{Y})/\sqrt{2}, \tag{2}$$

and $e^3(\mathrm{x})$ is a tangent direction to a *Principal Null Congruence (PNC)*, which is determined by the form[1]

$$e^3_\mu dx^\mu = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv, \tag{3}$$

via function $Y(\mathrm{x})$, which is controlled by *the Kerr theorem*, [3, 20, 21, 22].

The twisting lightlike rays of the Kerr PNC are focussing in the equatorial plane $\cos\theta = 0$, at the Kerr singular ring, $r = 0$, approaching it tangentially. As a result, the aligned with Kerr PNC metric and the KN electromagnetic potential,

$$A_\mu = -P^{-2} Re \frac{e}{(r + ia\cos\theta)} e^3_\mu, \tag{4}$$

concentrate near the Kerr ring and form a closed lightlike gravitational waveguide ,[12] playing the role of a closed string which may carry excitations in the form of the lightlike traveling waves [23, 24].

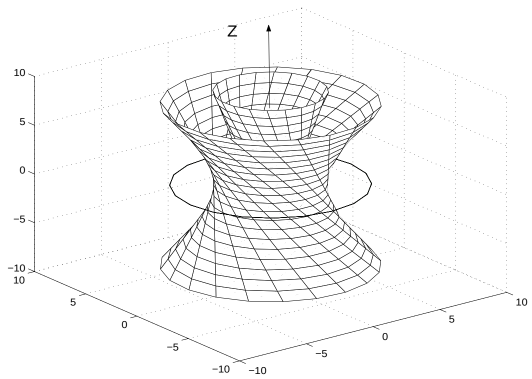

Figure 1: Twistor null lines of the Kerr congruence are focused on the Kerr singular ring, forming a twosheeted spacetime branched by closed string.

Treatment of the Kerr ring as a closed string was supported by the studies of the fundamental string solutions to low-energy string theory, [25]. There was obtained deep parallelism between the singular solutions of general relativity and the field solutions to low-energy string theory, [26]. In 1992 Sen obtained two important solutions to low-energy string theory: a) solution for fundamental heterotic string [27], and b) analog of the Kerr solution to low-energy string theory [28]. Then, it has been shown in [29] that the structure of electromagnetic field and metric around the Kerr singular ring in the solution a) is the same as that in the fundamental heterotic string solution b), for exclusion of the twovalued character of the fields which is caused by the twosheeted structure of the over-rotating Kerr space-time.

[1]Here $\zeta = (x + iy)/\sqrt{2}$, $\bar{\zeta} = (x - iy)/\sqrt{2}, u = (z + t)/\sqrt{2}$, $v = (z - t)/\sqrt{2}$, are the null Cartesian coordinates, r, θ, ϕ are the Kerr oblate spheroidal coordinates, and $Y(\mathrm{x}) = \mathrm{e}^{\mathrm{i}\phi} \tan\frac{\theta}{2}$ is a projective angular coordinate. The used signature is $(- + ++)$.

Twosheetedness of the Kerr geometry created a parallel line of investigation (Keres-Isreal-Hamity), which was evaluated in the López model of the bubble-source [7], where the singular region $r < r_e = e^2/2m$, was excised and the source accepted the form of a rigidly rotating ellipsoidal membrane, or bubble with a flat interior.

Regular source of the KN solution. Finally, in [11] the bubble source of the KN solution was generalized to a solitonic field model of a domain wall which interpolates smoothly between the external KN background and a false vacuum state inside the bubble. Gravitational singularity of the KN solution turns out to be suppressed by *a supersymmetric vacuum state forming by the Higgs field inside the bubble*, while the regularized em field formed a closed string on the boundary of the bubble.

Along with this closed string, the KN geometry contains also a *complex open string*, [13], which appears in the initiated by Newman complex representation of Kerr geometry, [30]. This string gives an extra dimension θ to the stringy source ($\theta \in [0, \pi]$), resulting in its extension to López membrane source [7, 11]. A superstring counterpart of this extension is the transfer from superstring theory to 11-dimensional M-theory and $M2$-brane, [15].

The Kerr theorem determines the shear free null congruences with tangent direction (3) by means of the solution $Y(\mathrm{x})$ of the equation $F(T^A) = 0$, where $F(T^A)$ is an arbitrary holomorphic function in the projective twistor variables $T^A = \{Y, \quad \lambda^1 = \zeta - Yv, \quad \lambda^2 = u + Y\bar{\zeta}\}$.

Using the Cartesian coordinates x^μ, one can rearrange variables and reduce generating function $F(T^A)$ to the form $F(Y, x^\mu)$, which allows one to represent solution of the equation $F(T^A) = 0$ in the form $Y(x^\mu)$.

Function $F(Y, x^\mu)$ for the Kerr and KN solutions is to be quadratic in Y,

$$F = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu), \tag{5}$$

and the equation $F = 0$ represents a *quadric* in the projective twistor space $\mathbf{CP^3}$. If determinant $\Delta = (B^2 - 4AC)^{1/2}$ is not degenerated, it defines the complex radial distance [20, 22]

$$\tilde{r} = -\Delta = -(B^2 - 4AC)^{1/2}. \tag{6}$$

The quadratic case is explicitly resolved and yields two solutions

$$Y^{\pm}(x^\mu) = (-B \mp \tilde{r})/2A, \tag{7}$$

which allows one to restore two PNC by means of (3). One can easily obtain from (5) and (7) that the used in the metric (2) and the em potential (4) complex radial distance $\tilde{r} = r + ia\cos\theta$ may also be determined from the Kerr generating function by the relation

$$\tilde{r} = -dF/dY. \tag{8}$$

Therefore, the Kerr singular ring, $\tilde{r} = 0$, is formed as a caustic of the Kerr congruence, $dF/dY = 0$.

As a consequence of the Vieta's formulas, the quadratic in Y function (5) may be expressed via the solutions $Y^{\pm}(x^\mu)$ in the form

$$F(Y, x^\mu) = A(Y - Y^+(x^\mu))(Y - Y^-(x^\mu)). \tag{9}$$

3. Complex Kerr geometry and an open complex string

One can see that the complex radial distance $\tilde{r} = r + ia\cos\theta$ takes in the Cartesian coordinates the form

$$\tilde{r} = \sqrt{x^2 + y^2 + (z + ia)^2}, \tag{10}$$

and therefore, the scalar component of the vector potential (4) may be obtained from the Coulomb solution $\phi(\vec{x}) = e/r = e/\sqrt{x^2 + y^2 + z^2}$ by a complex shift $z \to z + ia$, or by the shift of its singular point $\vec{x}_0 = (0, 0, 0)$ in complex region $\vec{x}_0 \to (0, 0, -ia)$. The complex shift was first considered by Appel in 1887 [31], who noticed that the Coulomb solution, being invariant solution to the linear Laplace equation with respect to a real shifts of its origin $\vec{x} \to \vec{x} + \vec{a}$, should also be invariant with respect to the complex shift. In spite of triviality of this procedure from complex point of view, it yields very nontrivial consequences in the real section, in particular, the singular point of the Coulomb solution $\vec{x}_0 = (0, 0, 0)$ turns into singular ring $x^2 + y^2 + (z + ia)^2 = 0$ (intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and plane $z = 0$), which becomes the branch line of the space into two sheets.

The obtained by Newman linearized form of the complex retarded-time construction, acquires exact meaning in the Kerr-Schild class of metrics, [23, 22, 20]. The KN solution in the KS form is generated by a *complex source propagating along a straight Complex World Line (CWL)*

$$x_L^\mu(\tau_L) = x_0^\mu(0) + u^\mu \tau_L + \frac{ia}{2}\{k_L^\mu - k_R^\mu\}, \tag{11}$$

where $u^\mu = (1, 0, 0, 1),\ k_R = (1, 0, 0, -1),\ k_L = (1, 0, 0, 1)$ and $\tau_L = t_L + \sigma_L$ is a complex retarded-time parameter. Index L labels it as a Left structure, and we should also add a complex conjugate Right structure

$$x_R^\mu(\tau_R) = x_0^\mu(0) + u^\mu \tau_R - \frac{ia}{2}\{k_L^\mu - k_R^\mu\}. \tag{12}$$

Therefore, *from complex point of view* the Kerr and Schwarzschild geometries are equivalent and differ only by a trivial complex shift. The nontrivial twisting structure of the Kerr geometry appears on the *real slice*, which for the Kerr solution goes aside of the center of solution. Complex shift turns the Schwarzschild radial directions $\vec{n} = \vec{r}/|r|$ into twisted directions of the Kerr congruence, Fig.1.

Complex open string It was obtained [13, 18] that the complex world line $x_0^\mu(\tau)$, parameterized by complex time $\tau = t + i\sigma$, represents really a two-dimensional surface which takes an intermediate position between particle and string. The corresponding "hyperbolic string" equation [18], $\partial_\tau \partial_{\bar\tau} x_0(t,\sigma) = 0$, yields the general solution

$$x_0(t,\sigma) = x_L(\tau) + x_R(\bar\tau) \tag{13}$$

as sum of the analytic and anti-analytic modes $x_L(\tau), \quad x_R(\bar\tau)$, which are not necessarily complex conjugate. For each real point x^μ, the parameters τ and $\bar\tau$ should be determined by a complex retarded-time construction. Complex source of the KN solution corresponds to two *straight* complex conjugate world-lines,(11),(12). Contrary to the real case, the complex retarded-advanced times $\tau^\mp = t \mp \tilde r$ may be determined by two different (Left or Right) complex null planes, which are generators of the complex light cone. It yields four different roots for the Left and Right complex structures [22, 20]

$$\tau_L^\mp = t \mp (r_L + ia\cos\theta_L) \tag{14}$$
$$\tau_R^\mp = t \mp (r_R + ia\cos\theta_R). \tag{15}$$

The real slice condition determines relation $\sigma = a\cos\theta$ with null directions of the Kerr congruence $\theta \in [0,\pi]$, which puts restriction $\sigma \in [-a, a]$ indicating that *the complex string is open*, and its endpoints $\sigma = \pm a$ may be associated with the Chan-Paton charges of a quark-antiquark pair. In the real slice, the complex endpoints of the string are mapped to the north and south twistor null lines, $\theta = 0, \pi$, see Fig.3.

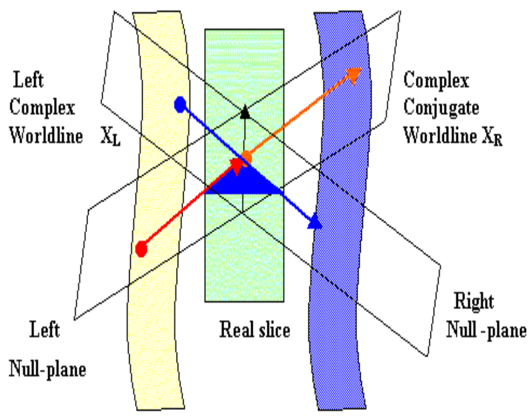

Figure 2: The complex conjugate Left and Right null planes generate the Left and Right retarded and advanced roots.

Orientifold projection. The complex open string boundary conditions [13] require the *worldsheet orientifold* structure [15, 17] which turns the open string in a closed but folded one. The world-sheet parity transformation $\Omega : \sigma \to -\sigma$ reverses orientation of the world sheet, and covers it second time in mirror direction. Simultaneously, the Left and Right modes are exchanged. The projection Ω is combined with space reflection $R : r \to -r$, resulting in $R\Omega : \tilde{r} \to -\tilde{r}$, which relates the retarded and advanced folds $R\Omega : \tau^+ \to \tau^-$, preserving analyticity of the world-sheet. The string modes $x_L(\tau), \quad x_R(\bar{\tau})$, are extended on the second half-cycle by

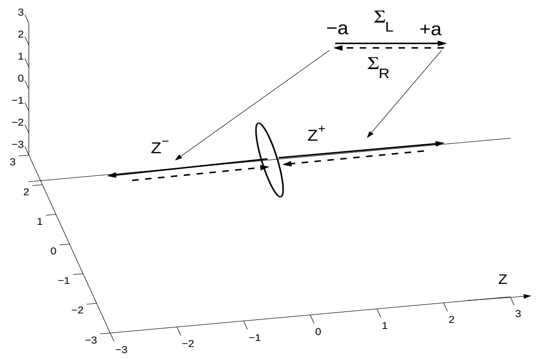

Figure 3: Ends of the open complex string, associated with quantum numbers of quark-antiquark pair, are mapped onto the real half-infinite z^+, z^- axial strings. Dotted lines indicate orientifold projection.

the well known extrapolation, [15, 17]

$$x_L(\tau^+) = x_R(\tau^-); \quad x_R(\tau^+) = x_L(\tau^-), \tag{16}$$

which forms the folded string, in with the retarded and advanced modes are exchanged every half-cycle.

The projection $\mathcal{T} = R\Omega$ sets parity between the positive Kerr sheet determined by the Right retarded time and the negative sheet of the the Left advanced time. It allows one to escape the anti-analytical Right complex structure, replacing it by the Left advanced one, and the problem is reduced to self-interaction of the retarded and advanced sources determined by the time parameters $\tau^\pm$. The presented in Fig.2 diagram shows a crossing symmetry the four roots $\tau^\pm$ and $\bar{\tau}^\pm$, for the complex retarded time, which allows one to replace the Right complex conjugate retarded-time structure $x_R(\tau^-)$ by the antipodal Left advanced-time structure $x_L(\tau^+)$, and works only in terms of the Left complex structures, omitting the index 'L'.

4. Calabi-Yau twofold from the Kerr theorem

The Kerr and KN solutions are stationary solutions of the Einstein-Maxwell field equations. In the KS formalism [3] they are characterized by a constant Killing direction K^μ, which corresponds to invariance of the metric with respect to the action of the operator $\hat{K} = K^\mu \partial_\mu$, and the condition of stationarity $\hat{K} g^{\mu\nu} = 0$ requires stationarity of the congruence $\hat{K} e^3 = 0$, which implies $KY = \hat{K}\bar{Y} = 0$. Killing direction K^μ may be expressed via parameters of the CWL, $x_0^\mu(\tau)$, as follows, $K^\mu = \partial_\tau x_0^\mu(\tau)$, and the coefficients A, B, C turn out to be functions of the coordinates $x_L^\mu(\tau^-)$ and 4-velocity of the CWL , $u^\mu(\tau^-) = \dot{x}_L^\mu(\tau^-) \equiv \partial_\tau x_L^\mu(\tau)|_{\tau^-}$, as functions of the parameter τ^-. However, it has been shown in [20] that explicit dependence from τ^- drops out for the stationary KS solutions. Excitations of the strings breaks stationarity of the KS solutions, and there appears radiation [32, 33] which should create a recoil. As a result, there appears explicit dependence of the KS solutions on the complex time parameter τ, and the generating functions $F(\tau^-)$ and $F(\tau^+)$ turn out to be independent. Consequently, parameters of the Kerr in-going congruence A_{in}, B_{in}, C_{in}, determined by τ^- becomes independent from parameters $A_{out}, B_{out}, C_{out}$, determined by τ^+, and should be considered as independent congruences generated by independent sources. Each of these sources produces a twosheeted KS geometry, and the formal description of the resulting four-folded congruence should be based on a multi-particle version of the Kerr theorem which corresponds to multi-sheeted twistorial structure of the KS geometry, [34]. In particular, the retarded and advanced pieces of the world line, $x_L^\mu(\tau^-)$ and $x_L^\mu(\tau^+)$, will generate two independent functions, F^+ and F^-, and create two different twistorial manifolds determined by two-particle version of the Kerr theorem. The corresponding two-particle function

$$F^{(2)}(T^A, x_L^+, x_L^-) = F^+(T^A, x_L^+) F^-(T^A, x_L^-) \tag{17}$$

represents the associated with the world-sheet parity, $\tau_L^- \leftrightarrow \tau_L^+$. two-point function formed as product of the functions F^+ and F^- corresponding to the retarded, $x_L(\tau^-)$, and advanced, $x_L(\tau^+)$, complex sources. The both factors, F^+ and F^- are quadratic in T^A, and each of the partial equations $F^+ = 0$ (or $F^- = 0$) generates a *quadric* in the projective twistor space CP^3 corresponding to the usual two-sheeted structure of the stationary KS geometry. The 'product' manifold, determined by the equation $F^{(2)}(T^A, x_L^+, x_L^-) = 0$, corresponds to a four-fold described as *a quartic* in the projective twistor space CP^3, which is the Calabi-Yau (complex) twofold, or the well-known $K3$ surface used in diverse models of the string compactification and also by generalization of superstring theory to M-theory,[15, 17]. We obtain that dynamical generalization of the Kerr geometry requires splitting of the complex source of Kerr geometry into independent retarded and advanced components described by the orientifold parity of the world-sheet. Orientifold appears as an stringy analog of the discussed by De Witt and Breme, [35] 'bi-tensor' fields, which are classical predecessors of the Feynman propagator.

5. Discussion

Connections between black holes and elementary particles are now commonly accepted. The Kerr solution plays in this respect especial role, since it represents a rotating black hole solution, metric of which may be consistency matched with gravitational field of spinning elementary particles. The charged Kerr-Newman solution has gyromagnetic ratio $g = 2$, as that of the Dirac electron [2, 3], and therefore, the KN gravitational field with the observable parameters of the electron: spin, mass, charge and magnetic momentum field corresponds to the electron background. However, due to extremely high spin/mass ratio of the spinning particles, this background is an over-rotating KN solution without horizons, and therefore, we arrive to principal conclusion that *black holes cannot be relevant to spinning particles.* Instead, the relevant geometry should be over-rotating KN solution, which contains a stringlike topological defect in the metric. Therefore, KN background needs an additional regularization to flat metric required by quantum theory. This 4d procedure turns out to be parallel to the enhancon model of the string/M-theory unification.

Relations of the Kerr solutions with string theory were noticed long ago. In particular, the conjecture that the Kerr singular ring may be treated as a closed string was first discussed in 1975 [12], and the complex Kerr string was first considered in [13] (1993). Recently these strings were reobtained by Adamo and Newman in [37], by analysis of the complex structure of the asymptotically flat space-times, and they write emotionally "...It would have been a cruel god to have layed down such a pretty scheme and not have it mean something deep."

In this paper we present extra remarkable evidence of the striking parallelism between the Kerr geometry and superstring theory, namely, appearance of the Calabi-Yau twofold in the complex twistorial structure of the Kerr geometry. In the recent paper [24] we argued that this parallelism is not accidental, because gravity is a fundamental part of the superstring theory, and the Kerr-Schild gravity, being based on twistor theory, displays also some additional inherent relationships with superstring theory. Roots of that are apparently related with underlying twistor theory, which provided the base for conformal invariance and the lightlike structure of the holographic projection, [33]. In fact the complex KS structure represents a complex version of the holographic projection, performed by complex null planes from the complex bulk on the real "screen-boundary".

In many respects the Kerr-Schild gravity resembles the twistor-string theory, [36, 38, 39], which is also four-dimensional, based on twistors and related with experimental particle physics. On the other hand, the complex Kerr string has much in common with the N=2 critical superstrings [18, 17, 40]. It is also related with twistors and has the complex critical dimension two which corresponds to four real dimensions and indicated that N=2 superstring may lead to four-dimensions. However, signature of the N=2 string may only be (2,2) or (4,0), which caused the obstacles for embedding of this string in the space-times with Minkowskian signature. Up to our knowledge, this trouble was not resolved so far, and the initially enormous interest to N=2 string seems to be dampened. Meanwhile, embedding of the N=2 string in the complexified Kerr geometry is almost trivial task, and approach to a super-generalization is also almost evident [41]. It hints that stringlike structures of the real and complex Kerr geometry are not only analogues, but may reflect the underlying twistorial structure and dynamics of the N=2 superstring.

Along with wonderful parallelism with the standard superstring theory, the stringy system of the four-dimensional KN geometry displays very essential peculiarities. Recall that origin of the string theory is closely related with experimental physics. In the recent historical paper J. Schwarz write that the transfer to the modern multidimensional version of string theory and choice for the fundamental string length scale 10^{-33} cm (the Planck length instead of 10^{-13} cm) occurred by 1973-1975 as consequence of some crisis in development of string theory, see [42]. He points out that price for this step was very high, since as a result the : "...construction of a complete and realistic model of elementary particles ...appears to be a distant dream."

The described stringy structures of the 4d KS geometry show that complexification of the Kerr geometry serves an alternative to traditional compactification of higher dimensions. The supplementary Kaluza-Klein space is absent, however the role of compactification circle is played by the closed Kerr string based on the Kerr singular ring, which admits traveling waves, realizing a "compactification without compactification", [24]. The lightlike twistorial rays are tangent to the Kerr singular ring, showing that the Kerr ring forms a lightlike fundamental string of the heterotic type, which is similar to DLCQ circle of M(atrix) theory, [43].

We arrive at the conclusion that the desired consistency of the superstring theory with physics of the spinning particles should be based on the complexified and super-generalized over-rotating Kerr-Schild geometry, accompanied by embedding of the complex N=2 string in its complex structure.

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