On noncommutative corrections in a de Sitter gauge theory of gravity

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Abstract

We present noncommutative corrections in a de Sitter gauge theory of gravity obtained using an analytical procedure with GRTensorII under Maple that we conceived to be applied for different solutions. First, the gauge fields lead to the components of field strength tensor and to other tensors and scalars of de Sitter gauge theory of gravity over the commutative space-time. Following the Seiberg-Witten map and using recursive formulas, the corrections are followed until second order through the second part of analytical procedure. The noncommutative analogue of the metric tensor is presented for the mapped solutions.

1. Introduction

The physics at very short distances, namely at Planck scale, requires different approach. At this scale the gravity is expected to be unified with the other fundamental forces but the exact mechanism remains unknown. Since in quantum field theory the classical dynamical variables become noncommutative it is naturally to analyze noncommutative features of gravity at this scale. The description of space-time as noncommutative space-time modifies the structure of the gravitational field. We study how the noncommutativity of space-time deform, through noncommutative parameters, some solutions of a gauge theory of gravity. The study is realized with some new analytical procedures conceived with GRTensorII under Maple that we designed for the specific quantities of the gauge theory of gravity. In the first procedure, for a de Sitter gauge theory of gravity, we define the gauge fields and we calculate the field strength tensors. This procedure is applied for several solutions in the de Sitter gauge theory of gravity and, for a vanishing torsion analogue, the specific quantities lead to the invariant action that is equivalent to Einstein action. The second procedure, based on Seiberg-Witten map, is that for the noncommutativity of the space-time, where we define the noncommutativity parameter and through the $*$-product we calculate, recursively, the leading deformation terms for gauge fields and field strength tensors, following them until second order. We present the analogous metric tensor for several solutions.

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We work with the model of gauge theory of gravitation that has the de-Sitter (DS) group SO(4,1) (10 dimensional) [1] as local symmetry and as base manifold, the commutative 4-dimensional Minkowski space-time, endowed with spherical symmetry:

$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{1}$$

The 10 infinitesimal generators of DS group $M_{AB} = -M_{BA}$, A,B=0,1,2,3,5, can be identified with translations $P_a = -M_{a5}$ and Lorentz rotations $M_{ab} = -M_{ba}$, a, b = 0,1,2,3. The 10 corresponding (non-deformed) gauge fields (or potentials) are $\omega_\mu^{AB}(x) = -\omega_\mu^{BA}(x)$. They are identified with the four tetrad fields (the gauge field of translational generator), $\omega_\mu^{a5}(x) = e_\mu^a(x)$, and the six antisymmetric spin connection $\omega_\mu^{AB}(x) = -\omega_\mu^{BA}(x)$. The field strength tensor, associated with the gauge fields $\omega_\mu^{AB}(x)$, which takes its values in the Lie algebra of the DS group (Lie algebra-valued tensor) is [2]:

$$F_{\mu\nu}^{AB} = \partial_{[\mu}\omega_{\nu]}^{AB} + \omega_{[\mu}^{AC}\omega_{\nu]}^{DB}\eta_{CD}, \tag{2}$$

with $\eta_{AB} = diag(-1,1,1,1,1)$ and the brackets indicate antisymmetrization of indices. The field strength tensor can be separated in torsion and curvature tensors:

$$F_{\mu\nu}^{a} = \partial_{[\mu}e_{\nu]}^{a} + \omega_{[\mu}^{ab}e_{\nu]}^{c}\eta_{bc}, \tag{3}$$

$$F_{\mu\nu}^{ab} = \partial_{[\mu}\omega_{\nu]}^{ab} + \omega_{[\mu}^{ac}\omega_{\nu]}^{db}\eta_{cd} + 4\lambda^2 e_{[\mu}^{a}e_{\nu]}^{b}, \tag{4}$$

where λ is a real parameter. For $\lambda \to 0$ we obtain the ISO(3,1), i.e., the commutative Poincaré gauge theory of gravitation. Defining $g_{\mu\nu} = \eta_{ab}e_\mu^a e_\nu^b$, the scalar $F = F_{\mu\nu}^{ab}\bar{e}_a^\mu\bar{e}_b^\nu$, with $e_\mu^a\bar{e}_b^\mu = \delta_b^a$, and $e = det(e_\mu^a)$, the gauge invariant action associated with the gauge fields is $S = -\frac{1}{16\pi G}\int d^4xeF$. Although the action appears to depend on the non-diagonal ω_μ^{AB} it is a function on $g_{\mu\nu}$ only.

2. Gauge fields solutions in the commutative theory

For a point like source of mass m and constant electric charge Q we adopt [2] the following gauge fields:

$$\begin{gathered} e_\mu^0 = (A,0,0,0)\,, \quad e_\mu^1 = \left(0,\frac{1}{A},0,0\right), \\ e_\mu^2 = (0,0,rC,0)\,, \quad e_\mu^3 = (0,0,0,rC\sin\theta)\,, \end{gathered} \tag{5}$$

$$\begin{gathered} \omega_\mu^{01} = (-U(r),0,0,0)\,,\ \omega_\mu^{02} = \omega_\mu^{03} = (0,0,0,0)\,,\ \omega_\mu^{12} = (0,0,Y(r),0)\,, \\ \omega_\mu^{13} = (0,0,0,H(r)\sin\theta)\,,\ \omega_\mu^{23} = (S(r),0,0,-\cos\theta)\,, \end{gathered} \tag{6}$$

where A, U, Y, H, S are functions of 3D radius r. If the components of $F^a_{\mu\nu}$ vanish then the spin connection components, ω^{ab}_μ, are determined by tetrads e^a_μ and, with the supplementary condition C = 1, we obtain:

$$\begin{aligned}\omega^{01}_\mu &= (AA', 0, 0, 0)\,,\ \omega^{02}_\mu = \omega^{03}_\mu = (0, 0, 0, 0)\,,\ \omega^{12}_\mu = (0, 0, -A, 0)\,,\\ \omega^{13}_\mu &= (0, 0, 0, -A\sin\theta)\,,\ \omega^{23}_\mu = (0, 0, 0, -\cos\theta))\,. \end{aligned} \tag{7}$$

The solution of field equations for gravitational gauge potentials $e^a_\mu(x)$ with energy-momentum tensor for electromagnetic field [4] can be find as: $A^2 = 1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2 - \frac{Q^2}{r^2}$, with the λ parameter identified with the cosmological constant $4\lambda^2 = -\frac{\Lambda}{3}$.

In the case of equivalent Robertson-Walker metric, the particular ansatz [3] for gauge fields

$$\begin{aligned} e^0_\mu &= (N(t), 0, 0, 0)\,,\quad e^1_\mu = \left(0, a(t)/\sqrt{1-kr^2}, 0, 0\right),\\ e^2_\mu &= (0, 0, ra(t), 0)\,,\quad e^3_\mu = (0, 0, 0, ra(t)\sin\theta)\,, \end{aligned} \tag{8}$$

$$\begin{aligned} \omega^{01}_\mu &= (0, U(t,r), 0, 0)\,,\ \omega^{02}_\mu = (0, 0, V(t,r), 0)\,,\\ \omega^{03}_\mu &= (0, 0, 0, W(t,r)\sin\theta)\,,\ \omega^{12}_\mu = (0, 0, Y(t,r), 0)\,,\\ \omega^{13}_\mu &= (0, 0, 0, H(r)\sin\theta)\,,\ \omega^{23}_\mu = (0, 0, 0, -\cos\theta)\,, \end{aligned} \tag{9}$$

with the constant k and the functions U, V, W, Y, H of time t and 3D radius r, leads for $F^a_{\mu\nu} = 0$ to

$$\begin{aligned} \omega^{01}_\mu &= \left(0, -\dot a(t)/\sqrt{1-kr^2}, 0, 0\right),\ \omega^{02}_\mu = (0, 0, r\dot a(t), 0)\,,\\ \omega^{03}_\mu &= (0, 0, 0, r\dot a(t)\sin\theta)\,,\ \omega^{12}_\mu = \left(0, 0, \sqrt{1-kr^2}, 0\right),\\ \omega^{13}_\mu &= \left(0, 0, 0, \sqrt{1-kr^2}\sin\theta\right),\ \omega^{23}_\mu = (0, 0, 0, -\cos\theta)\,. \end{aligned} \tag{10}$$

The spin connection components ω^{ab}_μ are, therefore, determined by the tetrads e^a_μ. We imposed the supplementary condition N(t) = 1.

For a spinning source of mass m we adopt the four tetrad fields:

$$\begin{aligned} e^0_\mu &= \left(\sqrt{\frac{\Delta}{\Sigma}}, 0, 0, -\frac{a\sin\theta}{\sqrt{\Sigma}}\right),\quad e^1_\mu = \left(0, \sqrt{\frac{\Sigma}{\Delta}}, 0, 0\right),\\ e^2_\mu &= \left(0, 0, \sqrt{\Sigma}, 0\right),\quad e^3_\mu = \left(-a\sqrt{\frac{\Delta}{\Sigma}}\sin^2\theta, 0, 0, \frac{r^2+a^2}{\sqrt{\Sigma}}\sin\theta\right), \end{aligned} \tag{11}$$

with the common notation $\Sigma(r,\theta) = r^2 + a^2\cos^2\theta$, $\Delta(r) = r^2 + a^2 - 2mr$, where $a = \frac{J}{m}$ is the angular momentum per unit mass. From the general form of the six antisymmetric spin connection:

$$\begin{aligned}
\omega^{01}_\mu &= \left(C(r,\theta),0,0,B(r,\theta)\right),\ \omega^{02}_\mu = \left(Q(r,\theta),0,0,P(r,\theta)\right),\\
\omega^{03}_\mu &= \left(0,U(r,\theta),V(r,\theta),0\right),\ \omega^{12}_\mu = \left(0,W(r,\theta),Y(r,\theta),0\right),\\
\omega^{13}_\mu &= \left(E(r,\theta),0,0,H(r,\theta)\sin\theta\right),\ \omega^{23}_\mu = \left(S(r,\theta),0,0,R(r,\theta)\right),
\end{aligned} \tag{12}$$

where B, C, U, V, E, H, P, Q, R, S, W and Y are functions of 3D radius r and θ, we obtain in the case of $F^a_{\mu\nu} = 0$ the following:

$$\begin{aligned}
\omega^{01}_\mu &= \left(\tfrac{\Sigma\Delta'+2r\left(a^2\sin^2\theta-\Delta\right)}{2\Sigma^2},0,0,-a\sin^2\theta\tfrac{\Sigma\Delta'+2r\left(r^2+a^2-\Delta\right)}{2\Sigma^2}\right),\\
\omega^{02}_\mu &= \left(0,0,0,-\tfrac{\sqrt{\Delta}}{\Sigma}a\sin\theta\cos\theta\right),\ \omega^{03}_\mu = \left(0,-\tfrac{ar\sin\theta}{\Sigma\sqrt{\Delta}},\tfrac{a\cos\theta\sqrt{\Delta}}{\Sigma},0\right),\\
\omega^{12}_\mu &= \left(0,-\tfrac{a^2\sin\theta\cos\theta}{\Sigma\sqrt{\Delta}},-r\tfrac{\sqrt{\Delta}}{\Sigma},0\right),\ \omega^{13}_\mu = \left(0,0,0,-r\tfrac{\sqrt{\Delta}}{\Sigma}\sin\theta\right),\\
\omega^{23}_\mu &= \left(a\cos\theta\tfrac{r^2+a^2-\Delta}{\Sigma^2},0,0,-\cos\theta\tfrac{\left(r^2+a^2\right)^2-\Delta a^2\sin^2\theta}{\Sigma^2}\right).
\end{aligned} \tag{13}$$

Δ' is the first derivative of function Δ with respect to 3D radius r.

The non-null components of the strength tensor for a spinning source of mass m are:

$$\begin{aligned}
F^{01}_{01} &= U,\quad F^{20}_{01} = \frac{a\sin\theta}{2\sqrt{\Delta}}Z, & F^{13}_{01} &= \frac{a\sin\theta}{2\sqrt{\Delta}}X,\quad F^{23}_{01} = Z,\\
F^{10}_{02} &= a\sin\theta Z,\quad F^{02}_{02} = \frac{\sqrt{\Delta}}{2}X, & F^{13}_{02} &= \frac{\sqrt{\Delta}}{2}Z,\quad F^{23}_{02} = a\sin\theta T,\\
F^{03}_{03} &= \frac{\sin\theta\sqrt{\Delta}}{2}X,\quad F^{21}_{03} = \frac{\sin\theta\sqrt{\Delta}}{2}Z, & F^{03}_{12} &= \frac{\Sigma}{2\sqrt{\Delta}}Z,\quad F^{12}_{12} = \frac{\Sigma}{2\sqrt{\Delta}}X,\\
F^{01}_{13} &= a\sin^2\theta U, & F^{20}_{13} &= \frac{(r^2+a^2)\sin\theta}{2\sqrt{\Delta}}Z,\\
F^{13}_{13} &= \frac{(r^2+a^2)\sin\theta}{2\sqrt{\Delta}}X, & F^{23}_{13} &= a\sin^2\theta Z,\\
F^{10}_{23} &= \sin\theta(r^2+a^2)Z, & F^{02}_{23} &= \frac{a\sqrt{\Delta}\sin^2\theta}{2}X,\\
F^{13}_{23} &= \frac{a\sqrt{\Delta}\sin^2\theta}{2}Z, & F^{23}_{23} &= \sin\theta(r^2+a^2)T,
\end{aligned} \tag{14}$$

where

$$X = \frac{2a^2(\Sigma - 2r^2\sin^2\theta) - 2\Delta(\Sigma - 2r^2) - r\Delta'\Sigma}{\Sigma^3} - 8\lambda^2,$$

$$
\begin{aligned}
Z &= a\cos\theta \frac{4r(r^2+a^2-\Delta) - 2r\Sigma + \Delta'\Sigma}{\Sigma^3}, \\
U &= \frac{(\Sigma - 4r^2)(\Delta - a^2\sin^2\theta) - \Sigma^2 + 2r\Delta'\Sigma}{\Sigma^3} - 4\lambda^2, \\
T &= \frac{(r^2+a^2-\Delta)(\Sigma - 4a^2\cos^2\theta)}{\Sigma^3} - 4\lambda^2.
\end{aligned} \tag{15}
$$

3. Deformed gauge fields and noncommutative analogous metric tensor

The noncommutative gauge theory is described in terms of gauge fields (or potentials) $\hat{\omega}_\mu^{AB}(x,\Theta)$ and field strengths $\hat{F}_{\mu\nu}^{AB}$ that depend on deformation parameter of noncommutative coordinate algebra. We work with the canonical deformation of the Minkowski space-time based on $[x^\mu, x^\nu]_* = i\Theta^{\mu\nu}$ with real constant deformation parameter $\Theta^{\mu\nu} = -\Theta^{\nu\mu}$. In this space noncommutativity is realized with the (associative) Moyal $*$-product, $* = e^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu}$. Using the Seiberg-Witten map one expand the noncommutative gauge fields, that transform according to the noncommutative algebra, in terms of commutative gauge fields, that transform according to the commutative algebra. In powers of $\Theta^{\mu\nu}$, [5], (the (n) subscript indicates the n-th order in $\Theta^{\mu\nu}$) the tetrad fields, the spin connections and the field strength tensor are:

$$
\begin{aligned}
\hat{e}_\mu^a(x,\Theta) &= e_\mu^a(x) + e_{(1)}{}_\mu^a(x) + e_{(2)}{}_\mu^a(x) + \dots \\
\hat{\omega}_\mu^{ab}(x,\Theta) &= \omega_\mu^{ab}(x) + \omega_{(1)}{}_\mu^{ab}(x) + \omega_{(2)}{}_\mu^{ab}(x) + \dots \\
\hat{F}_{\mu\nu}^{AB}(x,\Theta) &= F_\mu^{AB}(x) + F_{(1)}{}_{\mu\nu}^{AB}(x) + F_{(2)}{}_{\mu\nu}^{AB}(x) + \dots
\end{aligned} \tag{16}
$$

The noncommutative field strength tensor (after the contraction to the group ISO(3,1)) being:

$$
\hat{F}_{\mu\nu}^{AB} = \partial_{[\mu}\hat{\omega}_{\nu]}^{AB} + \hat{\omega}_{[\mu}^{AC} * \hat{\omega}_{\nu]}^{DB}\eta_{CD}, \tag{17}
$$

for $F_{\mu\nu}^a = 0$ (the undeformed one) and with the usual brackets for the anticommutator, the first order expressions for the gauge fields are:

$$
e_{(1)}{}_\mu^a = -\frac{i}{4}\Theta^{\rho\sigma}\left(\omega_\rho^{ab}\partial_\sigma e_\mu^c + (\partial_\sigma\omega_\mu^{ab} + F_{\sigma\mu}^{ab})e_\rho^c\right)\eta_{bc}, \tag{18}
$$

$$
\omega_{(1)}{}_\mu^{ab} = -\frac{i}{4}\Theta^{\rho\sigma}\left\{\omega_\rho, \partial_\sigma\omega_\mu + F_{\sigma\mu}\right\}^{ab}. \tag{19}
$$

The first order of field strength tensors can be write [6]:

$$
F_{(1)}{}_{\mu\nu}^a = \partial_{[\mu}e_{(1)}{}_{\nu]}^a + \left(\omega_{(1)}{}_{[\mu}^{ab}e_{\nu]}^c + \omega_{[\mu}^{ab}e_{(1)}{}_{\nu]}^c + \omega_{[\mu}^{ab} *_{(1)} e_{\nu]}^c\right)\eta_{bc}, \tag{20}
$$

$$
F_{(1)}{}_{\mu\nu}^{ab} = \partial_{[\mu}\omega_{(1)}{}_{\nu]}^{ab} + \left[\omega_{(1)\mu}, \omega_\nu\right]^{ab} + \left[\omega_\mu, \omega_{(1)\nu}\right]^{ab} + \left[\omega_\mu\omega_\nu\right]_{*(1)}^{ab}. \tag{21}
$$

In order to be applied for the particular tetrad fields, all formulas are implemented in an analytical procedure conceived in GR Tensor II for Maple. Instead to present the second order terms for the gauge fields and field strength tensor as usually, they come in the particular form of analytical procedure that contain suggestive notations.

```
>grdef(`ev2{^a miu}:=(-I/8)*Tn{^rho^sigma}*(om1{^a^c rho}*
ev{^d miu,sigma}+om{^a^c rho}*(ev1{^d miu,sigma}
+F1a{^d sigma miu})+(I/2)*Tn{^lambda^tau}*
om{^a^c rho,lambda}*ev{^d miu,sigma,tau}+
(om1{^a^c miu,sigma}+F1ab{^a^c sigma miu})*ev{^d rho}+
(om{^a^c miu,sigma}+Fab{^a^c sigma miu})*ev1{^d rho}+
(I/2)*Tn{^lambda^tau}*((om{^a^c miu,sigma,lambda}+
Fab{^a^c sigma miu,lambda})*ev{^d rho,tau}))*eta1{c d}`);
```

```
>grdef(`om2{^a^b miu}:=(-I/8)*Tn{^rho^sigma}*
(om1{^a^c rho}*(om{^b^d miu,sigma}+Fab{^d^b sigma miu})+
(om{^a^c miu,sigma}+Fab{^a^c sigma miu})*om1{^d^b rho}
+om{^a^c rho}*(om1{^d^b miu,sigma}+F1ab{^d^b sigma miu})
+(om1{^a^c miu,sigma}+F1ab{^a^c sigma miu})*om{^d^b rho}
+(I/2)*Tn{^lambda^tau}*(om{^a^c rho,lambda}*
(om{^d^b miu,sigma,tau}+Fab{^d^b sigma miu,tau})
+(om{^a^c miu,sigma,lambda}+Fab{^a^c sigma miu,lambda})*
omega{^d^b rho,tau}))*eta1{c d}`);
```

```
>grdef(`F2a{^a miu niu}:= ev2{^a niu,miu}-ev2{^a miu,niu}+
(om{^a^c miu}*ev2{^d niu}-om{^a^c niu}*ev2{^d miu}+
om{^a^c miu}*ev{^d niu}-om2{^a^c niu}*ev{^d miu}+
om{^a^c miu}*ev1{^d niu}-om1{^a^c niu}*ev1{^d miu}+
(I/2)*Tn{^rho^sigma}*(om{^a^c miu,rho}*ev1{^d niu,sigma}
-om{^a^c niu,rho}*ev1{^d miu,sigma}+om1{^a^c miu,rho}*
ev{^d niu,sigma}-om1{^a^c niu,rho}*ev{^d miu,sigma})+
(-1/8)*Tn{^rho^sigma}*Tn{^lambda^tau}*
(om{^a^c miu,rho,lambda}*ev{^d niu,sigma,tau}
-om{^a^c niu,rho,lambda}*ev{^d miu,sigma,tau}))*eta1{c d}`);
```

```
>grdef(`F2ab{^a^b miu niu}:=
om2{^a^b niu,miu}-om2{^a^b miu,niu}+
(om{^a^c miu}*om2{^d^b niu}-om2{^a^c niu}*om{^d^b miu}+
om2{^a^c miu}*om{^d^b niu}-om{^a^c niu}*om2{^d^b miu}+
om1{^a^c miu}*om1{^d^b niu}-om1{^a^c niu}*om1{^d^b miu}+
(I/2)*Tn{^rho^sigma}*(om{^a^c miu,rho}*om1{^d^b niu,sigma}
-om1{^a^c niu,rho}*om{^d^b miu,sigma}+om1{^a^c miu,rho}*
om{^d^b niu,sigma}-om{^a^c niu,rho}*om1{^d^b miu,sigma})+
(-1/8)*Tn{^rho^sigma}*Tnc{^lambda^tau}*
(om{^a^c miu,rho,lambda}*om{^d^b niu,sigma,tau}
-om{^a^c niu,rho,lambda}*om{^d^b miu,sigma,tau}))*
eta1{c d}`);
```

The noncommutative analogue of the metric tensor is defined using the hermitian conjugate of tetrads: $\hat{g}_{\mu\nu} = \frac{1}{2}\eta_{ab}\left(\hat{e}^a_\mu * \hat{e}^{b*}_\nu + \hat{e}^b_\nu * \hat{e}^{a*}_\mu\right)$. The noncommutative scalar analog to F is $\hat{F} = \hat{e}^\mu_a * \hat{F}^{ab}_{\mu\nu} * \hat{e}^\nu_b$, where $\hat{e}^\mu_a$ is the $*$inverse of $\hat{e}^a_\mu$. The part of analytical procedure for these quantities can be read in [10].

For a point like source of mass m and constant electric charge Q with a r-θ noncommutativity we obtain

$$\begin{aligned}
\hat{g}_{00} &= -A^2 - \frac{A^2}{4}\left(2r\frac{A'^3}{A} + 2A'^2 + 5rA'A'' + AA'' + rAA'''\right)\Theta^2 + \mathcal{O}(\Theta^4) \\
\hat{g}_{11}(x,\Theta) &= \frac{1}{A^2} + \frac{1}{4}\frac{A''}{A}\Theta^2 + \mathcal{O}(\Theta^4) \\
\hat{g}_{22}(x,\Theta) &= r^2 + \frac{1}{16}\left(A^2 + 12rAA' + 4r^2(4A'^2 + 3AA'')\right)\Theta^2 + \mathcal{O}(\Theta^4) \\
\hat{g}_{33}(x,\Theta) &= r^2\sin^2\theta + \frac{1}{16}\left(\cos^2\theta + \right. \\
&\left. +4sin^2\theta(2rAA' - r\frac{A'}{A} + r^2AA'' + 2r^2A'^2)\right)\Theta^2 + \mathcal{O}(\Theta^4).
\end{aligned} \tag{22}$$

For arbitrary $\Theta^{\mu\nu}$, the deformed metric is not diagonal even if the commutative one has this property [7], [8].

Working with r-t noncommutativity for the Robertson Walker case we obtain a noncommutative metric tensor identical with [9] and for r-θ noncommutativity the noncommutative metric tensor is:

$$\begin{aligned}
\hat{g}_{00} &= -1 + \Theta^2\frac{a\ddot{a}r}{16(1-kr^2)}(3 - 7kr^2 - 4r^2\dot{a}^2) + \mathcal{O}(\Theta^4) \\
\hat{g}_{11} &= \frac{a^2}{1-kr^2} - \Theta^2\frac{a^2\left(\dot{a}^2(5-kr^2(1+kr^2)) + 4k(1-kr^2)\right)}{16(1-kr^2)^3} + \mathcal{O}(\Theta^4) \\
\hat{g}_{22} &= r^2a^2 + \Theta^2\frac{a^2}{16}\left(\frac{3kr^4(\dot{a}^2+k)+1}{1-kr^2} - 26r^2(\dot{a}^2+k)\right) + \mathcal{O}(\Theta^4) \\
\hat{g}_{33} &= r^2a^2\sin^2\theta + \Theta^2\frac{a^2}{16}\left(5\cos^2\theta + \right. \\
&\left. + \frac{r^2\dot{a}^2(20kr^2+4r^2\dot{a}^2-9)+4(4k^2r^4-3kr^2+1)}{1-kr^2}\sin^2\theta\right) + \mathcal{O}(\Theta^4) \\
\hat{g}_{01} &= \Theta^2\frac{a^2\dot{a}\ddot{a}r}{16(1-kr^2)^2}(3kr^2 - 2) + \mathcal{O}(\Theta^4)
\end{aligned} \tag{23}$$

If we treat the noncommutative analogue of the metric tensor as a standard metric tensor we can examine the deformed space time for different scale factor in this case of constant noncommutative parameter.

4. Conclusions

Noncommutative deformations of general relativity solutions through the gauge theory formalism for gravity were the aim of the analytical procedure construction. The analytical procedure with GRTensorII under Maple conceived, allows to analyze the influence of noncommutative parameter choice on noncommutative analogue of the metric tensor and further to fit it to find valuable interpretations of noncommutative corrections. Being based on recursive formulas, the procedure can be extended. The corrections are followed until second order for tetrad fields, spin connections, field strength tensor, scalar F and thus for the invariant action, but, in the paper are presented only for noncommutative analogue of the metric tensor. The corresponding deformed metric reveals the modified structure of gravitational field: in the case of black holes and in the case of isotropic homogeneous Robertson-Walker space-time of the (commutative) gauge theory of gravitation. When we treat it as standard metric tensor, for non-rotating black-hole it can be observed that the spherical symmetry is brooked, while for the Robertson-Walker the isotropy (with respect to one world line).

In this paper we focused on the technical specific details of noncommutative corrections computation for the chosen cases without many details about the noncommutative theory and about physical interpretations.

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